Fountain rainbows

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We present the first measurements of radiance spectra of rainbows. The bows on two sunny days (3 and 6 June 2008) were produced by the fountain in the Parque de las Ciencias, Granada, Spain, that consists of a rectangular perimeter of 40 spray nozzles. Optical thickness of the spray from each nozzle was approximately 0.5. Spectral purity of the primary bow was highest for orange and blue, reaching values of 23% and 7%, respectively, while skylight 90° from the Sun had a color purity of 34% (on 6 June). The secondary bow had much lower color purity with red absent because the regions around the bows and in Alexander's dark band were pale blue. The narrow sickle shape of the chromaticity curves for the primary bows and the absence of supernumerary bows indicated that the drop radius was between 0.2 and 0.4 mm. © 2008 Optical Society of America

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1. Introduction

Bright rainbows seen against dark backgrounds can appear stunning but, as Raymond Lee has shown, represent relatively poor color standards [1]. Lee performed digital analysis of color photographic slides [2] of several rainbows and found that color purity and gamut was quite small for even the brightest and most vividly colored of the bows, with values for the gamut below 2%. The intrinsic value of the bow's luminance and color was estimated by (1) subtracting the luminance of the background at each wavelength assuming it to be equal to that in Alexander's dark band just outside the primary bow and (2) calculating the difference of the luminance in the primary bow at maximum and minimum polarization. Without background lighting the spectral purity and gamut of the bows did increase considerably, but the gamut remained below 25%.

Radiance, contrast, and color purity of rainbows depend (1) on the spectra of drop sizes and shapes and (2) on the ambient conditions, which include optical thickness of the rain shaft and its directly illuminated portion, lighting of the background and foreground, and distance from the rain shaft [3,4]. Color purity increases as drop size increases so long as the drops remain spherical, which implies that the drops are less than about 2 mm in diameter. Color purity also increases as the background lighting decreases. Both color purity of the bow and the contrast with the light on either side of the bow attain maximum values when the optical thickness of the illuminated part of the rain shaft, $\tau_R \approx 0.25$ when $B_B = 0.01$ and $\tau_R \approx 1.0$ when $B_B = 0.25$. Color purity of the bows, which span the color spectrum in an angular width of roughly 2° for the primary and 3° for the secondary, is inherently reduced by spreading over the finite width of the Sun (0.53°). Measured purity is further reduced when the instrument aperture has a finite angular width (0.25° for our apparatus).

Measurements of spectral radiance derived from slides of rainbows rather than the bows themselves

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have high spatial resolution but poor spectral resolution, being confined to red-green-blue (RGB) values. Recapturing detailed spectral information from photographs can be done but only after proper calibration of the camera with a spectroradiometer. This has been done for skylight at times far from sunset but has not been done for the rainbow [5,6].

Measuring natural rainbows with spectroradiometers also present serious difficulties. First, measurements must be made one at a time. Thus, in the time needed to produce a complete set of measurements, the bows and general lighting, which are often quite ephemeral, may well change. Second, radiometers have angular resolution so coarse (typically around 2°) that a single measurement could span most of the width of the primary bow. Finally, there may be some difficulty in determining exactly where the spectroradiometer has pointed.

These difficulties can be largely overcome with rainbows produced by fountains. This converts the rainbow from a wild, often rapidly varying natural phenomenon to a partially controlled experiment. In place of spatial scanning across the rainbow, spectral measurements made at uniform time intervals in the same direction allow the Sun to do the work of moving the bow, since the Sun moves across the sky at almost exactly $15°\,h^{-1}$. Furthermore, the effective angular radius of the spectroradiometer can then easily be reduced enough to measure almost monochromatic light beams from the rainbow by placing it within a pinhole camera, or a long tube with a small hole at the end. The greater sensitivity and degree of control even allows analysis of the secondary bow.

The fountain we chose for this experiment was either by chance or on purpose designed ideally to produce bright double bows. Whereas many fountains shoot jets of water that subsequently break into drops that may vary widely in size and optical thickness, the fountain at the Parque de las Ciencias in Granada, Spain, shoots water from a series of 40 equal nozzles, each of which creates drops at its source. In addition, when wind speed is low, the optical thickness of the spray appears to be almost constant with time along any given line. Finally, even though the spray is situated in direct sunlight under a clear sky, the direct background is shaded by an overhanging deck, and therefore is relatively dark.

In the analysis that follows we used the fountain of the Parque de las Ciencias as our rainbow source. We present our measurements of the spectral radiance of the primary and secondary bows with an angular resolution of 0.25° on two almost calm and almost cloudless sunny mornings and compare these with theoretical results of a rainbow model.

2. Experimental Design

The experimental setup is shown in Fig. 1. Radiance was measured by a SpectraScan PR-650 spectroradiometer at 4 nm intervals from 380 to 780 nm

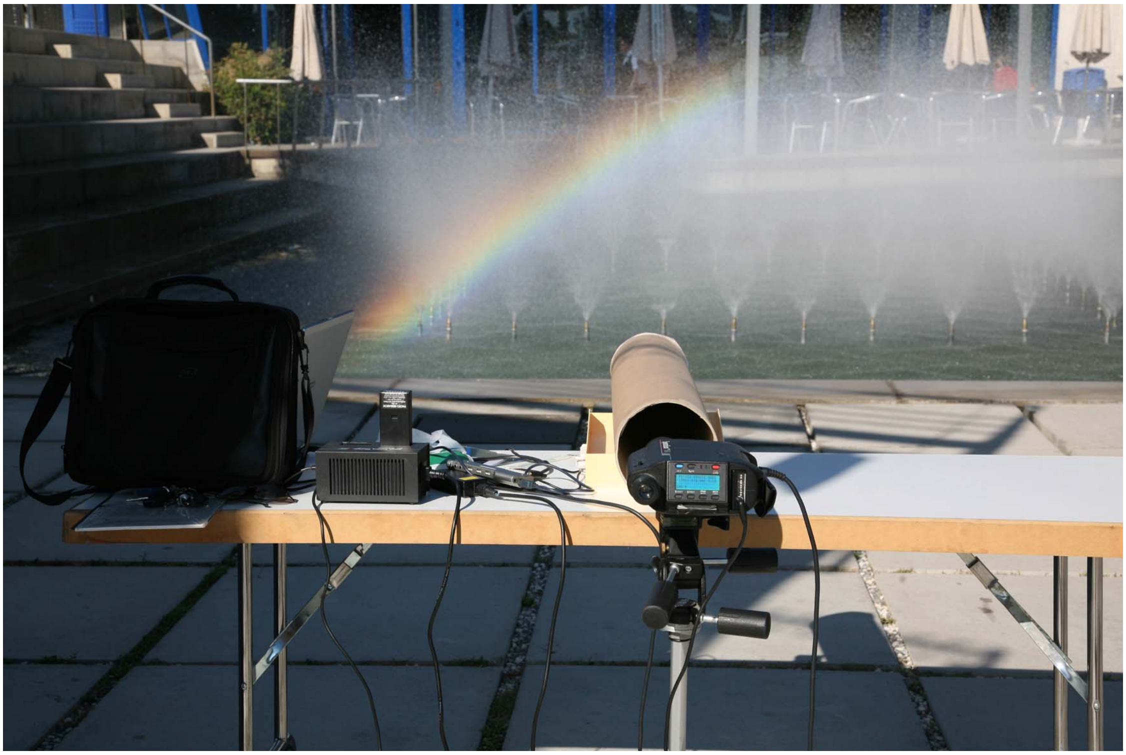

Fig. 1. (Color online) Experimental setup with orientation for measuring the primary bow on 6 June 2008, showing the fountain with 40 nozzles along its perimeter, the primary bow, and the spectroradiometer pointing into the tube below the bright overhang on the deck across the pool. Each stone on the ground is about 1 m^2. Optical thickness of the spray from each nozzle was estimated from the degree of blurring of the deck to be about 0.5.

within a cone of angular diameter 2°. The instrument has a stated error of less than 4% for radiance, a spectral accuracy of 2 nm, and CIE (Commission Internationale de l'Eclairage) 1931 colorimetric errors, $x < 0.001$, $y < 0.001$ for a 2856 K blackbody (CIE standard illuminant A) [7].

The spectroradiometer was aimed down a long cylindrical tube 0.15 m in diameter and 1.04 m long. The far end was sealed except for a hole with diameter 0.4 cm. Thus the angular width of the hole at the distance of the spectroradiometer was 0.25°. At the beginning of each experiment, the apparatus was aimed about 2° below the inside of the left side of the primary and secondary bows (i.e., a clock angle of about 11:00), and 25 to 30 measurements were taken at intervals of 1 min, or 0.25° apart.

The fountain used in the experiment is located at the Parque de las Ciencas in Granada, Spain. As mentioned above, it consists of 10 equally spaced nozzles on each side along the perimeter of a square about 5 m long. Each nozzle produced a spray of drops at the source. The maximum height of the drops was about 3 m. The drops were large enough that no supernumerary was visible and a secondary bow was produced. This means the drop radii were larger than about 0.2 mm. Alexander's dark band was also clearly marked. When dark objects were viewed through the line of 10 nozzles, they could not be seen, so that the approximate optical thickness was 5. Thus the spray from each nozzle had an optical thickness of about 0.5. This is adequate to produce a bright rainbow.

The background consisted of a building behind an overhanging deck. The floor of the deck was about 1 m above the level of the pool at the opposite end from the fountain. The deck jutted out about half a meter from the cement wall below, which consequently was shaded. The apparatus was aimed at the darkest part of the cement wall below the deck on the opposite side of the pool and about 0.25 m above the water surface. One important limitation of the experiment was imposed by the encroaching shadow of the observation tower, which progressively reduced the sunlit area of the spray as measurements of both bows and especially the secondary bow proceeded. This forced us to locate and aim the apparatus further to the right side of the fountain for the secondary bow. The orientations of the apparatus and locations of the shadows are shown in Fig. 2.

Measurements were made on 3 and 6 June 2008 in Granada (37.118 °N, 3.378 °W, altitude 680 m). On 3 June the primary bow was measured from about 0830 UTC to 0857 UTC and the secondary from 0904 UTC to 0924. On 6 June the primary was measured from 0818 to 0842 and the secondary from 0847 UTC to 0920 UTC. On 3 June, thin cirrus crept across the Sun during a few of the measurements for the primary and somewhat thicker cirrus for a few of the measurements of the secondary. On 6 June the sky was clear. It was almost calm on 3 June, and there were only light winds on 6 June so that spray was distributed almost symmetrically from each of the nozzles at all times.

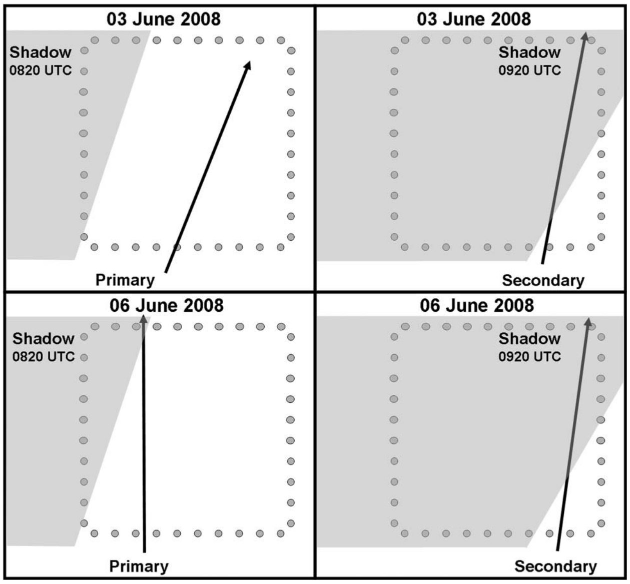

Fig. 2. Orientation of the spectroradiometer (arrows) for each of the experiments. The right edges of the gray shaded regions indicate the right edge of the shadows for each of the four experiments. Gray arrows indicate that the shadows covered all parts of the fountain further to the left. All times are UTC.

In all cases the apparatus was aimed so that it intercepted spray from two nozzles. Even when the shadow fell across most of the area of spray, the spray from at least the front row of nozzles remained sunlit. Thus the effective optical thickness of the sunlit part of the spray was about 1 for most measurements of the primary bow and at least 0.5 for measurements of the secondary.

3. Results of the Experiments

Personal observations of rainbows are subjective and hence of limited value, but they do constitute a starting point for analysis. The primary fountain bows appeared to both of us to be brighter and more vividly colored than many natural rainbows we have seen and with a good range of colors, but certainly not as spectacular as those rare intense rainbows we have seen beneath thick thunderclouds with dark backgrounds. The secondary fountain bows appeared distinct but were weak with much less range or purity of color, but they appeared to cover the range from red to blue. Alexander's dark band appeared unmistakably, with the apparent contrast of brightness across the primary bow much greater than across the secondary.

Radiance integrated over the visible light spectrum (here chosen as $400 \le \lambda \le 700$ nm) is shown for points traversing the primary bow on 3 June and for the primary and secondary bows on 6 June in Fig. 3, where the points of maximum purity of the blue and red bands of the bows are indicated. Radiance increases from the inside of the primary

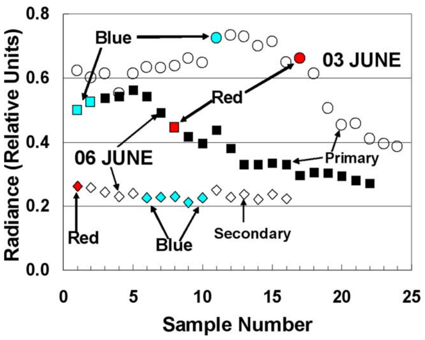

Fig. 3. (Color online) Total radiance (relative units) for the primary fountain bow of 3 June 2008 (circles) and the primary bow (solid squares) and secondary bow (hollow diamonds) of 6 June 2008. Points with greatest red and blue color purity are indicated by arrows and color fills.

bows to a peak in the bow a few data points (0.25 to 0.75°) beyond the point of highest blue purity and thereafter decreases into Alexander's dark band where it reaches values of about half of the peak. Radiance of the secondary bows also experiences a general decrease with time, most likely because of the encroaching shadow mentioned above. However, it does increase a few points just after (above) the broad blue peak of the secondary bow.

Spectral radiances for four different points across the primary bow of 3 June are shown in Fig. 4. The points inside and outside the bow have a similar spectral shape, but the radiance is much lower at all wavelengths in Alexander's dark band. Spectra for the samples with the highest red and blue color purity are quite different, with the red peaking in a broad wavelength band from 560 to 680 nm and the blue peaking more sharply in the wavelength band from 465 to 490 nm. Several features of the measured spectra agree qualitatively with simple rainbow theory. Radiance is greater for the blue than the red part of the bow because near the inside of the bow all colors contribute while mainly red contributes to the outside of the bow (aside from background and sky light). Indeed, radiance in the red maximum at wavelengths less than about 500 nm is little more than in Alexander's dark band, where the difference is probably due to multiple scattering.

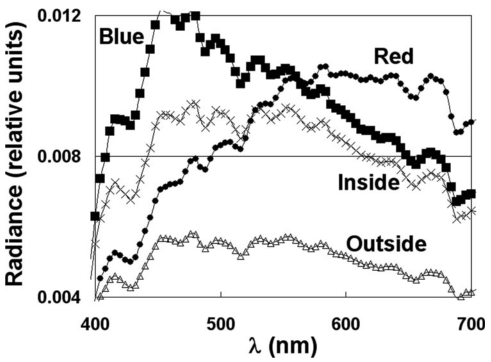

Fig. 4. Radiance spectra of primary fountain bow on 3 June 2008. The four samples are taken from inside the bow (#1, x's), the blue peak (#11, solid squares), the red peak (#17, solid circles), and in Alexander's dark band (#24, hollow triangles).

The sequence of color and color purity across the primary bows is shown in the 1931 CIE chromaticity diagrams of Fig. 5 for 3 June and Fig. 6 for 6 June. Both curves have a similar sickle shape and similar magnitudes. Peak color purity is higher for yellow to orange (23% and 17%) than for blue (7%) and much lower for green and purple. The chromaticity curves for modeled geometric optics rainbows (Fig. 7), are also sickle shaped but have a much greater excursion toward the red, as is characteristic of larger spherical drops [8]. Considering further that the fountain did not produce visible or detectable supernumerary bows, the Lee diagram shows that the likely spray drop radii fell in the range between 0.2 and 0.4 mm. The somewhat higher color purity of the red peak on 3 June was probably caused by changing the orientation of the apparatus so that the optical thickness of the illuminated part of the fountain was lower on 6 June.

The chromaticity diagrams for the secondary bows (not shown) both started at or below the bows near the neutral point and drifted counterclockwise to blue with maximum color purity about 7%. Outside the bow, the lighting remained pale blue with color purity about 3%. Most spectral measurements inside Alexander's dark band failed due to inadequate lighting. The pale blue color outside the secondary bows was almost certainly due to light reflected from the clear, blue sky, and was also the reason that red was not measured at the inside of the secondary bows. The measured absence of red contradicted our visual impression of pale reddish color at the inside of the secondary bows, which was most likely a consequence of the contrast of colors.

4. Summary and Conclusions

In this paper we presented the first measurements of radiance spectra of rainbows by measuring the bows produced on two sunny days (3 and 6 June 2008) by the fountain of the Parque de las Ciencias in Granada, Spain. The fountain consists of a perimeter

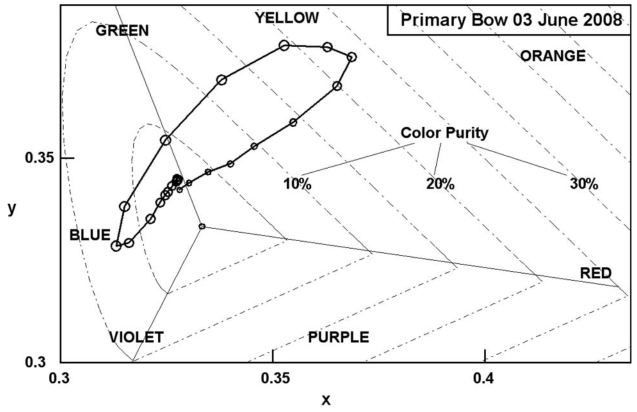

Fig. 5. 1931 CIE diagram showing color purity for the primary fountain rainbow of 3 June 2008. The diameters of the hollow circles are proportional to total radiance.

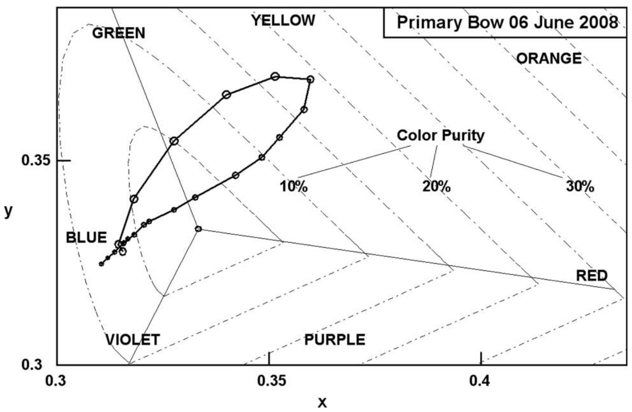

Fig. 6. Same as Fig. 5 but for the primary fountain rainbow of 6 June 2008.

lined by 40 equal spray nozzles, each of which creates drops at its source that shoot up to about 3 m and have an optical thickness of about 0.5. The primary and secondary bows were distinctly visible as was Alexander's dark band, and no supernumerary bows were observed or measured. The apparatus consisted of a spectroradiometer aimed down a 1 m long tube with an aperture 0.4 mm in diameter that created an effective angle of 0.25°. Measured radiance was lowest in Alexander's dark band (about half the maximum value) and greatest in the green and blue region of the primary bows. The CIE chromaticity diagrams for the primary bows had a narrow sickle shape which, combined with the absence of supernumerary bows, indicated that drop radius was in the range between about 0.2 and 0.4 mm. Maximum color purity was greatest for yellow-orange (23% and 17%) and for blue (7%) and minimum for green and purple. Color purity of the secondary bows was much lower with red not detected at the inner edge because the light of the region above the secondary bow was pale blue.

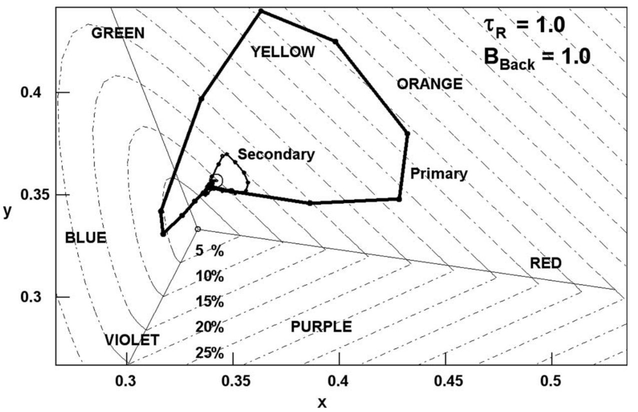

Fig. 7. Model geometric optics rainbow chromaticity curve for solar zenith angle, 75°, background brightness equal to clear skylight (= 1), optical thickness of the rain shaft, $\tau_R = 1$, and atmospheric turbidity, $\beta = 1.2$.

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References

1. R. L. Lee, "What are 'all the colors of the rainbow'?," Appl. Opt. **30**, 3401–3407 (1991).
2. R. L. Lee, "Colorimetric calibration of a video digitizing system: algorithm and applications," Color Res. Appl. **13**, 180–186 (1988).
3. S. D. Gedzelman and M. Vollmer, "Atmospheric optical phenomena and radiative transfer," Bull. Am. Meteorol. Soc. **89**, 471–485 (2008).
4. S. D. Gedzelman, "Simulating rainbows in their atmospheric environment," Appl. Opt. **47**, H176–H181 (2008).
5. M. A. López-Álvarez, J. Hernández-Andrés, J. Romero, and R. L. Lee, "Designing a practical system for spectral imaging of skylight," Appl. Opt. **44**, 5688–5695 (2005).
6. M. A. López-Álvarez, J. Hernández-Andrés, J. Romero, F. J. Olmo, A. Cazorla, and L. Alados-Arboledas, "Using a trichromatic CCD camera for spectral skylight estimation," Appl. Opt. **47**, H31–H38 (2008).
7. Photo Research, Inc., 9731 Topanga Canyon Place, Chatsworth, California 9131, USA.
8. R. L. Lee, "Mie theory, Airy theory and the natural rainbow," Appl. Opt. **37**, 1506–1519 (1998).