

## Measurements of sensitivity to simulated chromatic frequencies for normal and dichromatic observers

To cite this article: E Valero *et al* 1998 *J. Opt.* **29** 339

View the [article online](#) for updates and enhancements.

### You may also like

- [Training and development for informal science learning](#)  
Jane Randall Crockett
- [FM-to-AM conversion measurement for high power nanosecond lasers](#)  
Denis PENNINCKX
- [Problèmes du traitement optique de l'information: \(Considérations sur des systèmes mettant en oeuvre les hologrammes et les concepts de bandes spectrales et de ressemblance des formes\)](#)  
Jean-Charles Viénot, Jacques Duvernoy and Gilbert Tribillon

# Measurements of sensitivity to simulated chromatic frequencies for normal and dichromatic observers

E Valero, J A Garcia, J L Nieves and J Romero

Dpto de Óptica, Universidad de Granada, Campus Fuentenueva, s/n 18071 Granada, Spain

Received 3 March 1998, accepted 7 July 1998

**Abstract.** In recent years, the study of the human colour vision system in the chromatic-frequency domain has been a source of interesting results, concerning especially the spectral modulation sensitivity function (SMSF). Here, we present a detailed study of the mid-frequency range of this function for two normal observers and also the SMSF for two observers with colour-vision deficiencies (protanopes). The measurements have been obtained by simulating stimuli of sinusoidal spectral power distribution. Observers viewed metamers of these stimuli which were generated on a CRT monitor. The minimum found at 1.6 c/400 nm for normal observers is confirmed and some explanation is attempted by the calculation of the Fourier transform of the spectral response of colour-vision mechanisms. The results for dichromatic observers show a general loss of sensitivity with respect to the normal subjects, especially at certain frequencies.

**Keywords:** Colour vision, chromatic frequency, colour-vision deficiencies

## Mesures de la sensibilité à des fréquences chromatiques simulées pour des observateurs normaux et dichromatiques

**Résumé.** Pendant les dernières années, l'étude du système humain de vision de la couleur dans le domaine des fréquences chromatiques a donné des résultats très intéressants, spécialement en rapport avec la fonction de sensibilité à la modulation spectrale. Nous présentons ici une étude détaillée de l'intervalle des fréquences intermédiaires de cette analyse pour deux observateurs normaux et aussi la fonction de sensibilité à la modulation spectrale pour certains observateurs avec des différentes anomalies de la vision de la couleur (protanopes). Les mesures ont été obtenues au moyen de la simulation de stimuli de distribution spectrale d'énergie sinusoïdale. Les observateurs ont aperçu des métamères de ces stimuli produits par un moniteur en couleurs. La valeur minimale trouvée à 1.6 c/400 nm des observateurs normaux est confirmée, et on essaye de l'expliquer au moyen du calcul de la transformation de Fourier de la réponse spectrale des mécanismes de vision de la couleur. Les résultats des observateurs anomaux nous montrent une perte générale de sensibilité par rapport aux observateurs normaux, spécialement pour certaines fréquences.

**Mots clés:** Vision de la couleur, fréquences chromatiques, anomalies de la vision de la couleur

### 1. Introduction

Since Barlow noticed the possibilities offered by the study of human colour vision in the chromatic-frequency (first called comb-frequency) domain (Barlow 1982), a number of studies have attempted to formulate an overall idea of the behaviour of the human visual system in this domain

(Barlow *et al* 1983, Benzschawel *et al* 1986, Bonnardel and Varela 1991, Bonnardel *et al* 1996, Romero *et al* 1992, 1995a, b, 1997). The function known as the spectral modulation sensitivity function (SMSF) has been obtained using either a theoretical approach (Romero *et al* 1995a) or an experimental approach (Bonnardel and Varela 1991, Bonnardel *et al* 1996, Romero *et al* 1997). The SMSF

is the analogue in the chromatic-frequency domain to the contrast sensitivity function (CSF) and the temporal modulation transfer function (TMTF) in the spatial- and temporal-frequency domain.

To obtain this SMSF, it is necessary to study the visual system response to stimuli of spectral composition given by

$$E(\lambda) = E_0(1 + m \sin(fp(\lambda) + p_0)) \quad (1)$$

where  $f$  is the chromatic frequency that we express in  $c/400$  nm,  $p_0$  is the initial phase,  $m$  the relative amplitude (within  $[0, 1]$ ) and  $p(\lambda)$  is a linear transformation,

$$p(\lambda) = (0.9\lambda - 333) \text{ deg} \quad (2)$$

which transforms the spectral interval  $[370\text{--}770$  nm] into the phase interval  $[0\text{--}360^\circ]$ . Stimuli of the spectral power distribution given by expression (1) are compared, by means of a discrimination criterion, to stimuli of null modulation ( $m = 0$ ); the initial value of  $m$  ( $m = 1$ ) is then decreased until the stimulus cannot be discriminated from the null modulation one. The value of sensitivity for each frequency and initial phase is given by the inverse of the amplitude  $m$  of the threshold stimulus. The SMSF function is then obtained by representing, for each frequency, the maximum value of sensitivity (optimal envelope of the curves for different initial phases between  $0^\circ$  and  $360^\circ$ ).

When this is done theoretically (Romero *et al* 1995a), using MacAdam (1943) discrimination ellipses, a smooth SMSF is obtained, with a single peak at 1.8 or 2.0  $c/400$  nm, depending on the colour tolerance used, and a cut-off frequency between 5.5 and 8.0  $c/400$  nm. The form of the curve is very similar to that of a typical CSF. However, if the SMSF is obtained experimentally using observers with normal colour vision (Bonnardel and Varela 1991, Bonnardel *et al* 1996, Romero *et al* 1997) the results show a profile for the curve which is substantially less smooth. Bonnardel and Varela (1991) expressed their results for their three observers on a logarithmic scale, finding a slow increase of the sensitivity in the low-frequency range, until around 0.8  $c/300$  nm, and a rapid decrease from 1.55–1.8  $c/300$  nm to the cut-off frequency. In the mid-frequency range there was a minimum located at a different frequency for each observer. In this study, Bonnardel and Varela did not generate stimuli such as those of SPD given by equation (1); they made a square modulation of the SPD of the luminous source used.

Bonnardel *et al* (1996) obtained, by means of sinusoidally modulating the SPD of a xenon bulb and for one observer with normal colour vision, again the same slow increase of the SMSF until 0.97  $c/300$  nm, where the maximum value of the curve was reached, then a flat zone until 2.2  $c/300$  nm and a rapid decrease from this frequency onwards. When they represented the pessimal envelope (given by the minimum value of sensitivity for each frequency), the maximum value of the curve was reached at 1.23  $c/300$  nm (1.64  $c/400$  nm).

In our laboratory, we have measured the SMSF using a CRT colour monitor with a high-resolution graphics card as our stimulus generator (Romero *et al* 1997). We took sensitivity measurements for 41 chromatic frequencies

at equal intervals in the range 0.2–8.2  $c/400$  nm and eight initial phases between  $0^\circ$  and  $315^\circ$ . The method consisted basically in purity discrimination experiments in which simultaneous comparisons were made of a stimulus metamer of a flat SPD with another metamer of a SPD corresponding to equation (1). We progressively diminished the value of amplitude  $m$  of this second stimulus until the discrimination threshold was reached. The luminance of both stimuli was fixed throughout the experiment at 20  $\text{cd m}^{-2}$ .

Our results for two normal observers (Romero *et al* 1997) show features similar to those of previous studies—the sensitivity increased for low frequencies and decreased for higher frequencies until the cut-off was reached. However, we found for both observers a significant minimum at 1.6  $c/400$  nm, precisely where we could expect to find the maximum value of the SMSF. A second but less significant minimum also appeared at 0.8  $c/400$  nm.

The presence of this minimum at 1.6  $c/400$  nm has led us to attempt a more detailed study of the mid-frequency range, where other authors have found a certain range of variability in their measurements of the SMSF. We consider it necessary to confirm by further measurements the minimum at 1.6  $c/400$  nm. To accomplish this, we have extended the number of initial phases used from 8 to 16 and we have taken sensitivity measurements of sensitivity for 1.4, 1.6 and 1.8  $c/400$  nm. If this result is confirmed, we shall seek an explanation for it.

Measurements of the SMSF for observers with anomalous colour vision have been obtained recently by Bonnardel *et al* (1997). These kind of results are highly informative in order to establish a link between the study of the human colour vision in the chromatic-frequency domain and colour-vision models. For us, it was also useful to obtain them as a means of finding a possible explanation of the results for both types of observers, normal and anomalous, in the mid-frequency range. We used two anomalous observers for this part of the study.

## 2. Results

The experimental device and method used in the present work has been explained in detail elsewhere (Romero *et al* 1997). The test field is similar to that developed by Krauskopf and Gegenfurtner (1992). It consisted of four coloured circles. In three of them, we displayed the reference achromatic stimulus and in the other one the metamer of the modulated stimulus, varying its position randomly for each presentation. The observer was asked to locate the stimulus that appeared different from the other three. In our case, we did not use a staircase procedure as did the authors referred to above in order to measure the discrimination threshold, but rather we obtained it by a statistical fitting of the distribution of discrimination responses to the stimuli presented to compare with the equienergetic stimulus metamer. It can be deduced that for a given frequency and initial phase, the stimuli presented are located in the CIE 1931 chromaticity diagram in the line that connects the maximum modulation stimulus with the point ( $x = 0.333$ ,  $y = 0.333$ ).

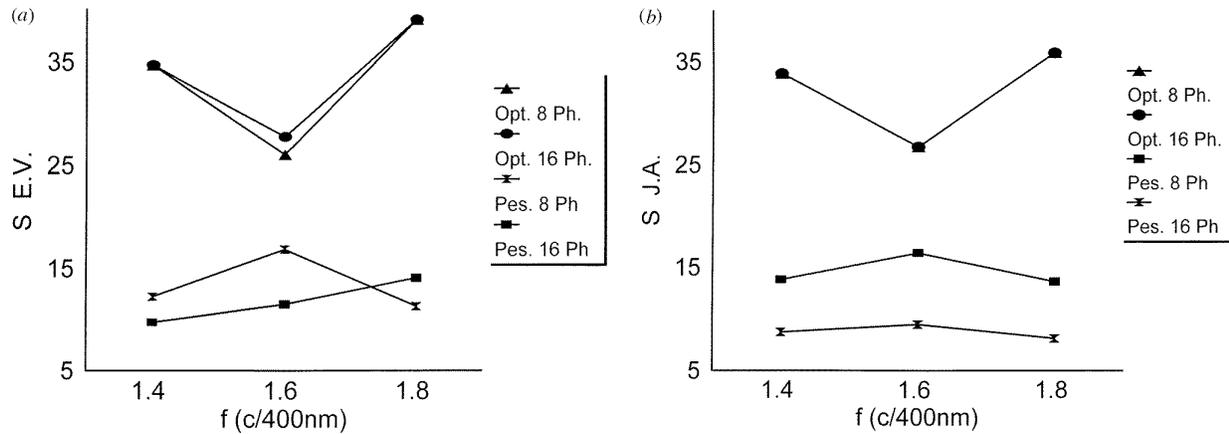


Figure 1. The optimal and pessimal curves for normal observers: (a) EV and (b) JA.

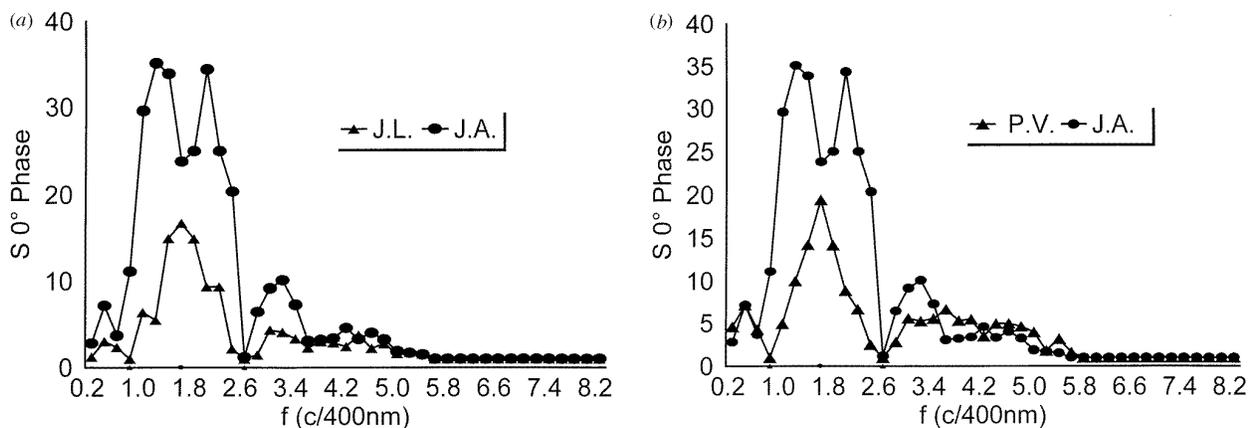


Figure 2. Comparison between (a) spectral modulation sensitivity of protanope observer JL and normal observer JA, (b) spectral modulation sensitivity of protanope observer PV and normal observer JA. The sensitivity was measured at 0° initial phase.

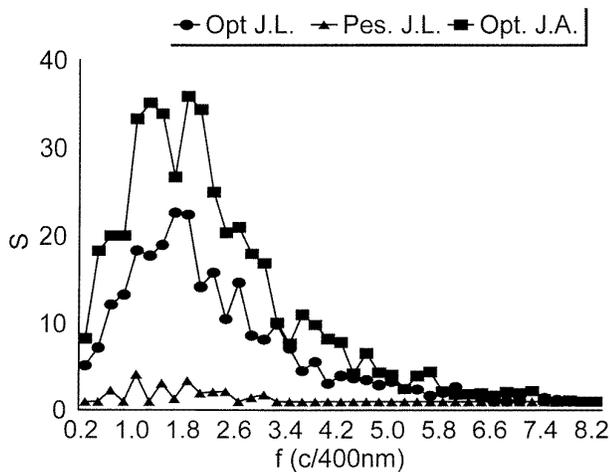
We have taken sensitivity measurements for the following initial phases: 30°, 60°, 120°, 150°, 210°, 240°, 300° and 330°. We had previously measured the sensitivity for the same two observers for initial phases of 0°, 45°, 90°, 135°, 180°, 225°, 270° and 315° (Romero *et al* 1997). The results for both normal observers are shown in figure 1, including the results obtained previously (Romero *et al* 1997), so we have used the data corresponding to 16 initial phases (eight of them being new measurements) instead of just eight initial phases (as in Romero *et al* 1997) in order to calculate the upper envelope for the three frequencies represented in figure 1. Therefore the minimum at 1.6 c/400 nm is confirmed when we extend the number of initial phases used to obtain the optimal curve. Regarding the non-optimal curve, there are small differences of sensitivity for the various frequencies, indicating that there is less variation in the sensitivity value for 1.6 c/400 nm when we change the initial phase than for the other two frequencies.

As stated above, we collected data for two observers with dichromatic colour vision, both of them protanopes (JL and PV). The diagnosis and classification of the anomalies was established using the Ishihara test and Heideberg anomaloscope (Nagel type). The observer JL completed the

sensitivity measurements for eight initial phases (0°, 45°, 90°, 135°, 180°, 225°, 270° and 315°), while PV completed just for one initial phase (0°). The range of chromatic frequencies studied was from 0.2 to 8.2 c/400 nm.

In figure 2, we show the results of sensitivity for both observers in the 0° initial phase, making comparisons to the sensitivity measured for the normal observer JA (Romero *et al* 1997). We find a general decrease in sensitivity for dichromatic subjects over the whole range of frequencies. However, a decrease in the cut-off frequency value is not present.

We also find significant differences in the shape of the curves between normal and dichromatic observers, who presented a maximum at 1.6 c/400 nm where the normal observers have a minimum. For 0° initial phase, this frequency corresponds to a tritanopic confusion line and this fact explains the results if we take into account that red-green dichromatic observers have better discrimination along this line than other lines that involve red-green discrimination. As we expected, at the frequencies nearest the red-green confusion lines, 0.8 and 2.6 c/400 nm for this initial phase, the sensitivity falls to the minimum possible value of 1.



**Figure 3.** Comparison between the optimal curves for the protanope JL and normal JA observers. Also shown is the pessimal curve for the protanope JL observer.

If we look at the optimal and pessimal curves for observer JL (figure 3), we find a maximum value for the optimal envelope at  $1.6 c/400 \text{ nm}$ , where the normal observers have a significant minimum. The loss of sensitivity is clear for the whole range of frequencies, but we can find no strong differences in the cut-off frequency. The pessimal envelope presents small variations in sensitivity.

The cut-off frequency obtained for JL is  $7.6 c/400 \text{ nm}$ . For the normal observers EV and JA, we previously measured a cut-off frequency of  $7.6$  and  $7.8 c/400 \text{ nm}$ , respectively. Thus, we did not find a lower cut-off frequency for dichromatic observers, in agreement with other authors (Bonnardel *et al* 1997, for two deuteranope observers). As the SMSF is the optimal envelope of the curves measured for the different initial phases, the cut-off frequency for this curve is the maximum of the cut-off frequencies obtained for each initial phase. If we calculate the mean of these cut-off frequencies, we find a value of  $6.4 c/400 \text{ nm}$  for JL, lower than  $6.8 c/400 \text{ nm}$ , which is the mean cut-off frequency for both normal observers. This is due to the fact that, for some initial phases ( $135^\circ$ ,  $270^\circ$  and  $315^\circ$ ), the cut-off frequencies for JL are appreciably lower than those for EV and JA, because for these initial phases the high-frequency region is quite near the red–green confusion line.

### 3. Discussion

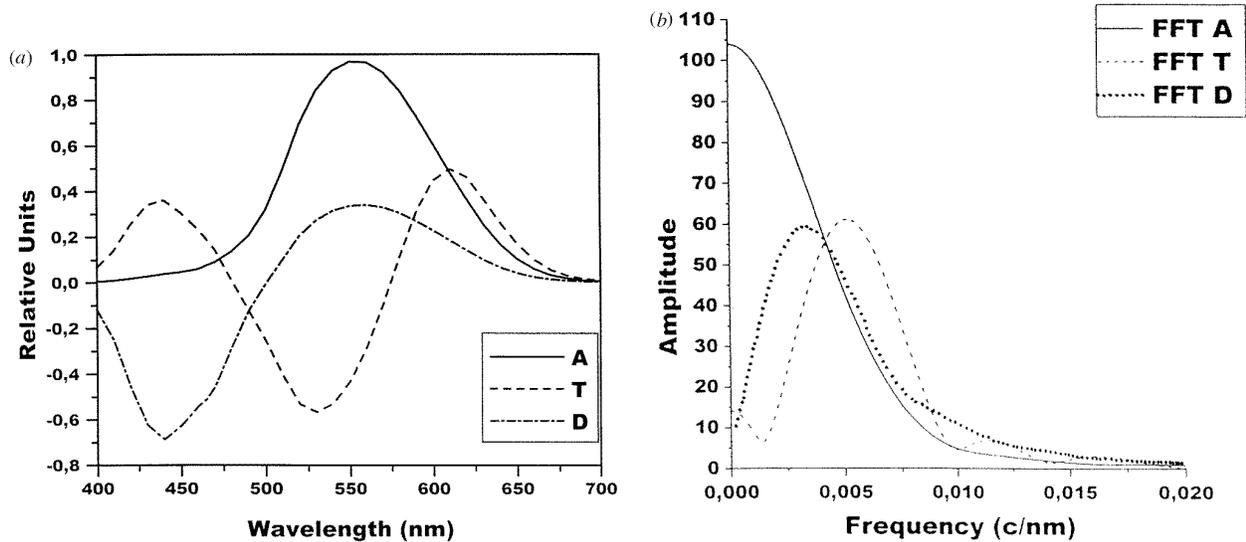
First, we will attempt to explain the minimum of sensitivity at  $1.6 c/400 \text{ nm}$  and the cut-off frequency value obtained in studying the response of the colour vision mechanisms in the chromatic-frequency domain. Barlow (1982) showed the Fourier transform of the L, M and S cone fundamentals obtained by Smith and Pokorny (1975). Barlow obtained smooth curves decreasing continuously from the null frequency value to  $0.015 c/\text{nm}$  ( $6 c/400 \text{ nm}$ ) for L and M,  $0.02 c/\text{nm}$  ( $8 c/400 \text{ nm}$ ) for S. These could be considered

cut-off frequencies for each fundamental, though we cannot regard them strictly as cut-off frequencies if we take into account that the fundamentals are not band-limited functions (they are defined only in a certain interval of wavelength). However, the value of the modulus for the Fourier transform of the fundamentals at these frequencies (and higher) is almost zero. We can see that the frequency cut-off value for the S fundamental is just slightly higher than the cut-off frequency measured for the two normal observers and the protanope. In particular, it is worth noting for this last observer, who, being of a red–green defective type, should have S cones that work normally, so that we can expect that his cut-off frequency will be limited by these cones, as happens with normal observers. For experiments leading to threshold points located further from the equienergetic stimulus, we should obtain lower cut-off frequencies, as we have shown by our theoretical calculations in a previous work, Romero *et al* (1995a).

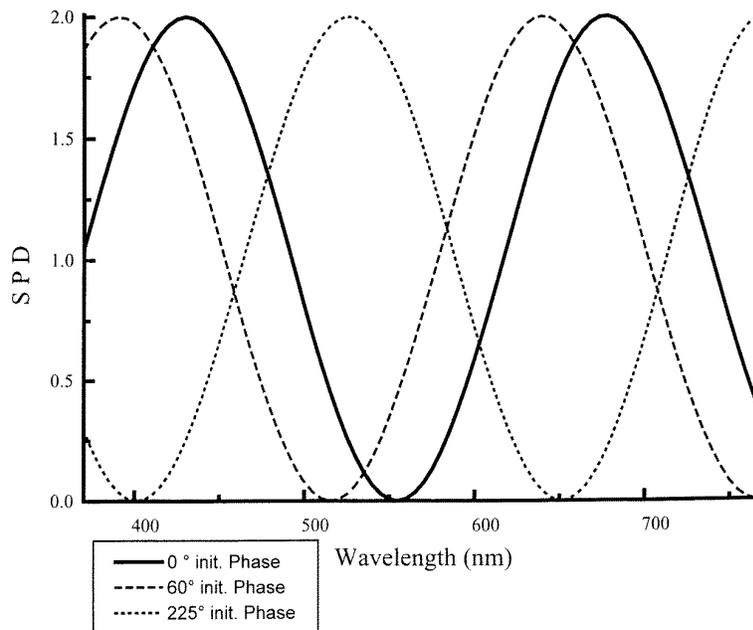
We have calculated the Fourier transform of the spectral response for A, T and D mechanisms of the model of Ingling and Tsou (1977) (figure 4). This is equivalent to the calculation of the optimal response of these mechanisms to a stimulus of type (1) with maximum modulation (Bonnardel *et al* 1997). As expected, we find cut-off frequencies of the same order as those obtained for L, M and S. In addition, there appear intermediate maxima for the D and T channels, located at  $0.003 c/\text{nm}$  ( $1.2 c/400 \text{ nm}$ ) and  $0.005 c/\text{nm}$  ( $2 c/400 \text{ nm}$ ). Between these two frequencies, we find the maximum value zone for our experimental sensitivity measurements. This interval of frequencies is also included in the nearly flat zone for the SMSF measured by Bonnardel and Varela (1991) and Bonnardel *et al* (1996). However, they found a wider range of frequencies in this zone of maximum values. This may be due to the fact that they do not work at constant luminance and thus the A mechanism can contribute somewhat to the shape of the curve, or, in the case of Bonnardel and Varela (1991), because the stimuli used are not strictly of a single frequency.

Figure 4(b) indicates that the Fourier transforms of mechanisms T and D have the same modulus at  $0.0415 c/\text{nm}$  ( $1.66 c/400 \text{ nm}$ ) and also the A mechanism reaches one-half of its maximum value. Therefore, the frequency at which we found the significant minimum for the SMSF,  $1.6 c/400 \text{ nm}$ , is almost where the three Fourier transforms are of the same value.

In our experiments, we have worked consistently at a constant luminance and thus it is quite curious that we have found the minimum just where the red–green and yellow–blue mechanisms' Fourier transforms have the same value. Taking into account that from the pessimal curve results we can deduce that for  $1.6 c/400 \text{ nm}$  there is less variation of sensitivity with phase than for the other contiguous frequencies studied, we can establish that the visual system is less sensitive when the responses of the colour mechanisms are balanced and the sensitivity depends more on the shape of the spectral power distribution observed than on its origin. In figure 5, we show the SPD for a stimulus of  $1.6 c/400 \text{ nm}$  and a number of initial phases. This curve is what we call opponent type, with



**Figure 4.** (a) Spectral response of the A, T and D channels (Ingling and Tsou 1977). (b) Fourier transform of spectral responses of A, T and D channels.



**Figure 5.** Spectral power distribution with a stimulus of  $1.6 \text{ c}/400 \text{ nm}$  for initial phases of  $0^\circ$ ,  $60^\circ$  and  $225^\circ$ .

two maxima and one minimum, resembling the shape of the T-channel spectral response, or else with two minima and one maximum.

The maximum of the SMSF for observer JL (protanope) was slightly shifted to higher frequencies with respect to the maximum of the Fourier transform of the D mechanism for protanopes (deduced from the Ingling and Tsou model), which is located at  $1.252 \text{ c}/400 \text{ nm}$ . This lack of coincidence between the two maxima might be due to the influence of the A-mechanism response. For this observer, we cannot assume that we were working at a constant brightness as is the case with normal observers. But the fact of using the same matching functions in order

to present the stimulus for dichromats and for normal observers is in our opinion quite correct, as suggested by Kaiser and Boynton (1996) in their recent book: 'One might expect therefore that observers who are missing one of the three photopigment classes would accept color matches made by normal observers who possess all three classes of photopigments. This prediction might follow because a match for all three types of receptors should remain a match for each member of the remaining pair if only one type is missing'.

Lastly, it is worth noting that for both normal observers the SMSF obtained was very similar to the envelope of the Fourier transform amplitude for mechanisms T and D

(figure 4 (b)), though the minimum is less significant in this envelope. We might also say that the chromatic-frequency response of the visual system to stimulus at a constant luminance is determined by the D-mechanism response in the low-frequency range, by the T-mechanism response in the high-frequency range and, when both mechanisms have the same response, the sensitivity falls appreciably as a consequence of this balanced situation.

### Acknowledgment

We would like to thank Professor H B Barlow for his kind and generous suggestions that have been the starting point of this work.

### References

- Barlow H B 1982 What causes trichromacy? A theoretical analysis using comb-filtered spectra *Vis. Res.* **22** 635–43
- Barlow H B, Gemperlein R, Paul R and Steiner A 1983 Human contrast sensitivity for comb-filtered spectra *J. Physiol.* **340** 50P
- Benzschawel T, Brill M H and Cohn T E 1986 Analysis of human color mechanisms using sinusoidal spectral power distributions *J. Opt. Soc. Am. A* **3** 1713–25
- Bonnardel V, Bellemare H and Mollon J 1996 Measurements of human sensitivity to comb-filtered spectra *Vis. Res.* **36** 2713–20
- Bonnardel V, Ruderman D L and Barlow H B 1997 Fast determination of the spectral modulation sensitivity function: a comparison between trichromats and deuteranopes *Colour Vision Deficiencies* vol 13 ed C R Cavonius pp 415–24
- Bonnardel V and Varela F 1991 A frequency view of colour: measuring the human sensitivity to square-wave spectral power distributions *Proc. R. Soc. B* **245** 165–71
- Ingling C R and Tsou B H P 1977 Orthogonal combination of three visual channels *Vis. Res.* **17** 1075–82
- Kaiser P K and Boynton R M 1996 *Human Color Vision* (Washington, DC: Optical Society of America)
- Krauskopf M and Gengenfurtner H 1992 Color discrimination and adaptation *Vis. Res.* **32** 2165–75
- MacAdam D L 1943 Specification of small chromatic differences in daylight *J. Opt. Soc. Am.* **33** 18–26
- Romero J, Garcia J A, Valero E and Nieves J L 1997 Measurements of the spectral modulation sensitivity function for two normal observers with CRT monitors *J. Opt.* **28** 190–8
- Romero J, Jiménez del Barco L and Hita E 1992 Mathematical reconstruction of color matching functions *J. Opt. Soc. Am. A* **9** 25–9
- Romero J, Nieves J L, García-Beltrán A and Hita E 1995a Analysis of colour-vision mechanisms in the chromatic frequency domain *J. Opt.* **26** 9–15
- Romero J, Nieves J L and García-Beltrán A 1995b Human processing of colour information in the chromatic-frequency domain *Vis. Res.* **35** 867–71
- Smith V C and Pokorny J 1975 Spectral sensitivity of the foveal cone photopigments between 400 and 500 nm *Vis. Res.* **15** 161–71