Fizeau fringes at home

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We propose an easy and inexpensive laboratory experiment to obtain equal-thickness optical interference using a flatbed scanner and a personal computer to produce Fizeau fringes. Valuable discussions about multiple-beam interference, Fresnel equations, the theory of partial coherence, and colorimetry arise naturally from the interference-fringe results.
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I. INTRODUCTION

We often come across Fizeau fringes outside the optical laboratory, for example, in an oil film floating on a wet street illuminated by sunlight, or on a soap bubble. In both cases beautiful interference color patterns are seen when a white light source is involved. Some of these patterns are the phenomena we normally mention to students to focus their attention on occurrences of equal-thickness interference in their daily environment. In this paper we present a simple experiment that students can do either in their own homes or in a computer laboratory to take the study of such phenomena one step further. The main advantages of this experiment are the controlled conditions, in that the visibility of each fringe pattern can be modified in real time with the scanner control software and that the fringes can be recorded for later discussion and analysis.

We have divided the experiment into two clearly different steps. In Sec. II we describe how to obtain stable Fizeau fringes with a flatbed scanner, which the students can easily do on their own. Subsequently in Sec. III we discuss the background that students should receive to understand the observed interference. This background information is not always available in a basic optics laboratory because of a lack of instruments, such as a spectroradiometer, for example, so we present the essential data needed to stimulate the hoped-for discussion.

The proposed experiment should be done after having discussed equal-thickness interference fringes (also known as Fizeau fringes, or sometimes contour fringes) and the Newton's ring experiment. We recommend that students should also have some knowledge of partial optical coherence. A basic grasp of colorimetry, while not essential, might help deepen the discussion of the topic. If the students have a background in multiple interferences alone, the proposed experiment will give them a practical application of their theoretical knowledge.

II. PUZZLED STUDENTS AT HOME

We start by asking the students to study how a flatbed scanner works and to analyze its different optical components.[1] After they have inspected the inside of the scanner, we follow this request with a short introduction on scanner control software, focusing on the image histogram (distribution of pixel intensity values) and the exposure adjustment toolbox. We then encourage the students to scan several times with neither transparency nor paper in the scanner. By choosing the automatic exposure adjustment, the scanner software will show the *best* image, a completely white one. We then let the students play with the software, allowing them to modify randomly the highlights, shadows, and midtone levels of the white image. After a few minutes, it is very likely that they will discover a strange, unexpected pattern such as the one shown in Fig. 1, instead of a completely white image. At this juncture the students will be puzzled, unless they are capable of applying their knowledge of multiple interference.

III. DISCUSSION PROCEDURE

The next step—and the teacher's main goal—is to initiate a discussion about the possible causes, in order to arrive at a deeper understanding of optical interference and optical coherence. To do this we ask students who have a basic knowledge of optics to follow these steps: (1) generate several interference fringes with the scanner, (2) identify the interference, (3) study the possible causes, and (4) use the theory of partial coherence to arrive at the most plausible explanation for the observed phenomena. We suggest that this order is most appropriate for our students, although our recommendation may be altered according to the students' background and laboratory resources.

A. Interference fringes?

The first question arising from an observation of the patterns is their very nature. Are they interference fringes? And if so, what kind of interference? Bearing in mind that the students have previously inspected the insides of the scanner, this question is probably the easiest for them to answer, if they possess only a background in interference.[2–4]

The object to be scanned is placed on the glass of the scanner, as shown in Fig. 2. We can assume that this glass is a plane-parallel, dielectric film. Does it cause the fringe patterns? If we assume that there is only one illumination angle (taking into account the position and size of the source below the glass), it is tempting to conclude that the equal-thickness (Fizeau's) fringes must be caused by an unevenness in the thickness of the glass. This might be the students' first and most intuitive reasoning.

Nevertheless, there are two arguments against this conclusion. Figure 1 shows that the unevenness in the thickness (estimated by counting the number of interference fringes) might be too high to be due only to a manufacturing defect. And the average thickness of the glass in our scanner, as measured previously by the students, is approximately 3.5 mm.[1] So for this kind of interference the light source must be highly monochromatic. In fact the coherence length of the

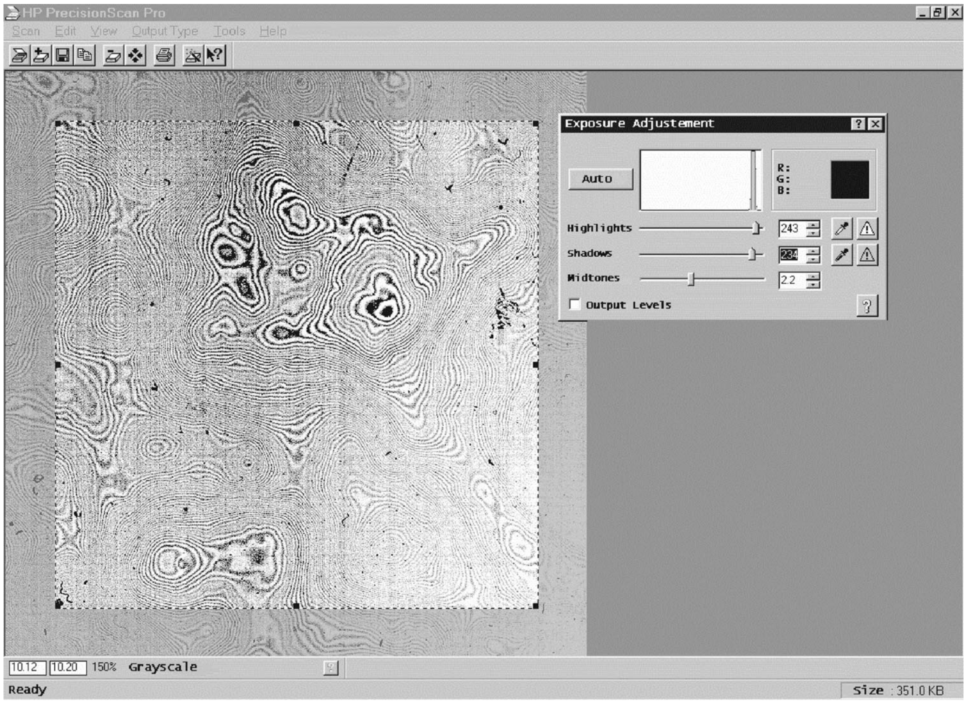

Fig. 1. An example of what is obtained with a flatbed scanner when we scan with neither transparency nor paper in the machine. The manipulation of the histogram (see the window at the upper right-hand corner), that is, modification of the highlights, shadows, and midtone levels, with the scanner software allows us to see a clear multiple interference pattern.

light, l_C, should be greater than $2nd$, where n is the glass refraction index and d is its thickness. If we believe that the interference fringes are produced by the glass and take $n \approx 1.5$, this condition would mean that the coherence length of the light source must be considerably more than 10.5 mm, which is a high value.

Thus the next step in our discussion is to consider the characteristics of the scanner's light source.

Fig. 2. Photograph of the HP Scanjet 6200C scanner.

B. Light source

What is the spectrum of the lamp? From the manufacturer's web site,[5] we note that the scanner has a custom cold-cathode, fluorescent lamp[1] with a long life expectancy. A typical relative spectrum of a cold fluorescent lamp is available in many books and web sites. To familiarize the students with such devices, we used a spectroradiometer[6] to measure the lamp spectrum and the result is shown in Fig. 3. The spectrum is plotted in the units most commonly used for visible spectra, that is, radiance per unit wavelength interval. An analysis of this spectrum will allow us to answer the question about the coherence length of the scanner lamp: Is the coherence length of the spectrum large enough to cause multiple interference in the glass?

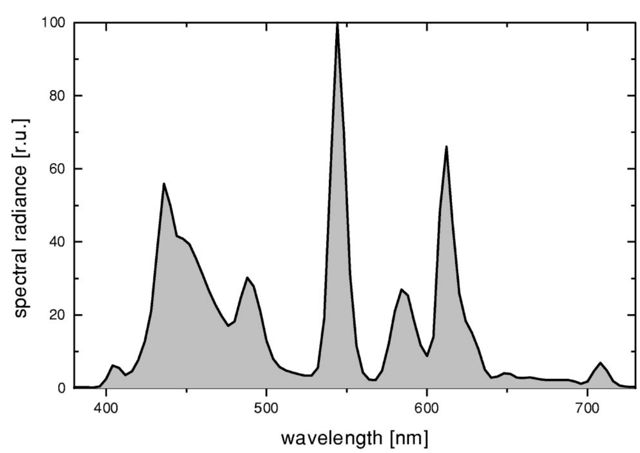

Fig. 3. Relative spectral radiance (in relative units) of the scanner's cool white fluorescent lamp.

At this juncture it is time to remind the students of some basic ideas of the theory of partial coherence. A parameter that can be used to specify the amount of temporal coherence in a light source is a characteristic time, known as the *coherence time*, τ_C, which specifies the time during which a source maintains its phase. For a Gaussian spectral distribution, τ_C is proportional to the reciprocal of the spectral width, $\Delta\upsilon$, of the source:

$$\tau_C=\frac{1}{\Delta\upsilon}. \tag{1}$$

The longer the coherence time, the more monochromatic the source. The coherence length of the source is related to the coherence time by

$$l_C=c\tau_C, \tag{2}$$

where c is the speed of light in vacuum.

For a Gaussian spectrum the coherence length can easily be obtained if the average wavelength, $\bar{\lambda}$, and the wavelength width, $\Delta\lambda$, of the spectrum are known, according to:

$$l_C=\frac{\bar{\lambda}^2}{\Delta\lambda}. \tag{3}$$

For a spectrum more complex than a simple Gaussian (see Fig. 3, for example), it is not so simple to estimate l_C or τ_C. We have included in the Appendix a brief description of the theory of partial coherence, including the equations needed to calculate the coherence time, and the results obtained for the scanner lamp we used.

So as not to complicate our discussion too much, we assume that the spectrum of the lamp can be described roughly by a Gaussian function, in order to use Eq. (1) or (3). We let the spectrum be Gaussian centered at 550 nm with a width of 250 nm (extending as it does from about 400 to 700 nm). Equation (3) gives a rough estimate of 1210 nm for the coherence length. This value gives a very small coherence length indeed, around two wavelengths of white light. Recall that in Sec. III A we required a coherence length of more than 10.5 mm if the glass were to be the cause of the interference pattern. Therefore, this rough estimate of the coherence length obliges us to reject the scanner glass as the source of the observed interference.

C. Air layer

The students are faced with a conundrum, and it is time to look more carefully at the components of the scanner. If we ask them to obtain a profile of the different optical media that the light travels through in the scanner, some of them might come up with the schematic shown in Fig. 4. They will realize that, no matter how well closed it is, there is an air layer between the glass and the white cover. The thickness of the air film must be variable because of the irregular contact between the cover and the glass, although this irregularity will be very small in comparison with the wavelength. This air film is the most plausible cause of the interference fringes: the air layer is the source of multiple-beam interference by amplitude division.

There are several features that confirm this hypothesis. First, the interference fringes are focused on the scanned image. So the fringes are formed very close to the glass, as might be expected if the air layer is small and if equal-thickness interference is taking place.[2–4] Second, the contrast

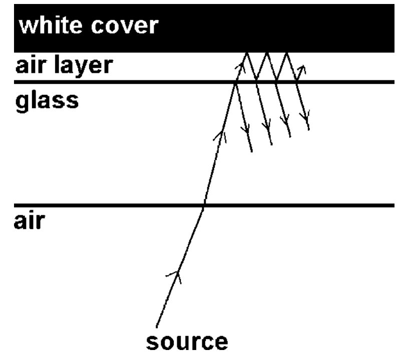

Fig. 4. Different layers of media in the scanner.

of the fringes is quite low as is evident from a quick look at a typical histogram of the scanner image (see Fig. 1). In fact, the histogram can be described as a narrow peak centered on the maximum intensity values. Can we predict the low contrast theoretically? Figure 5 shows the multiple-beam interference in the air layer. Let r' be the coefficient of reflection at the glass/air interface, r the coefficient of reflection at the air/white-cover interface, t the transmission coefficient at the glass/air interface, and t' the transmission coefficient at the air/glass interface. Note that the r and r' coefficients are not related because they refer to different interfaces (r for air/cover and r' for glass/air). From Fresnel's equations[2–4] we can relate the transmission coefficients at the glass/air and air/glass interface to the reflection coefficient at the same interface: $tt'=1-r'^2$. If there is no absorption, the amplitudes of the successive rays transmitted through the air layer are

$$ar',\, att'r,\, att'r^2,\, att'r^2r',\, att'r^3r'^2,\ldots, \tag{4}$$

as indicated in Fig. 5.

Each transmitted wave will have a constant phase difference relative to its neighbor, assuming normal incidence, given by

$$\delta=\frac{2\pi 2d}{\lambda_0}, \tag{5}$$

where d denotes the thickness of the air layer and λ_0 the wavelength in vacuum.

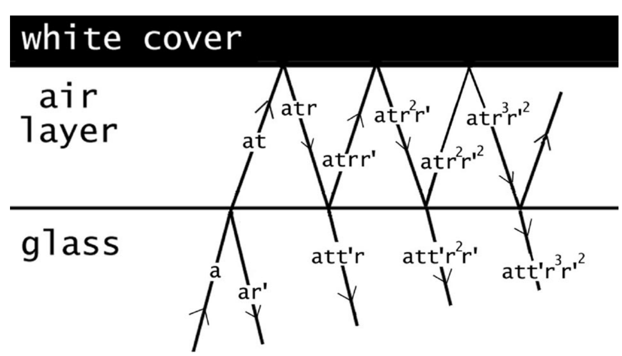

Fig. 5. Reflection of a wave at a plane-parallel air layer; r' is the coefficient of reflection at the glass/air interface, r the coefficient of reflection at the air/white-cover interface, t the transmission coefficient at the glass/air interface, and t' the transmission coefficient at the air/glass interface.

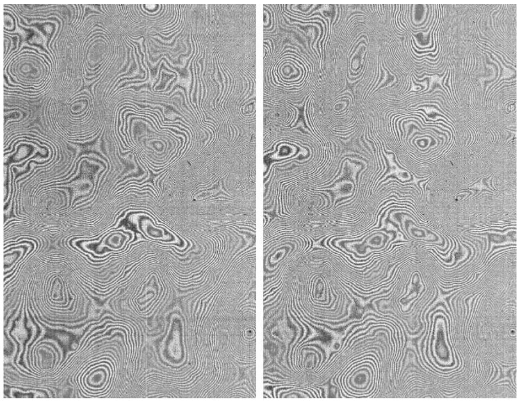

Fig. 6. Two interference fringe patterns obtained with the scanner. The difference between them is gentle pressure on the scanner cover caused by placing a book on the scanner.

It is an interesting exercise for the students to add up all the transmitted waves to calculate the intensity, I, of the transmitted light to determine the fringe visibility as

$$V=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}. \tag{6}$$

For a realistic estimation of the fringe visibility, we can assume as before that $n\approx 1.5$. By using a portable spectrophotometer,[7] we measured the spectral reflectance R of the white cover, obtaining a value of 80% across almost the entire visible spectrum. Both reflection coefficients, r and r', can be now estimated using Fresnel's equations.[2–4] From the relation $R=r^2$, the value of r is approximately 0.89. If we assume normal incidence, r' is just

$$r'=\frac{n-1}{n+1}\approx 0.2. \tag{7}$$

The fringe visibility calculated using these numerical values is about 8.1%, which is very low because the magnitude of the minimum intensity is comparable to the maximum intensity. So the fringe pattern is made up of intense fringes (maximum intensity) and slightly less intense fringes (minimum intensity). This is the reason the histogram is highly biased toward the maximum-intensity pixel values (Fig. 1). If we were not able to modify the highlights, shadows, and midtone levels of the image using the scanner software, we would not have detected the interference pattern.

The third fact that confirms the origin of the observed interference is that a small variation in pressure upon the white cover (by placing a book on it, for example) changes the fringe pattern because the thickness of the air layer is altered. An illustrative example is shown in Fig. 6, where the difference between the two patterns is due to gentle pressure on the cover. A more dramatic illustration is obtained by using nonflat objects. Figure 7 shows the effect of pressure from three fingertips and a roll of cellotape. The black areas in both cases reveal that the thickness of the air layer is $d=0$, corresponding to a minimum intensity fringe for all wavelengths simultaneously.

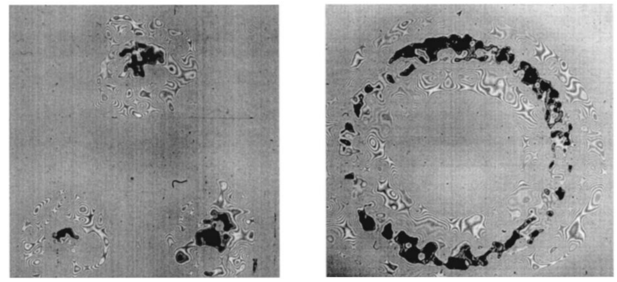

Fig. 7. Two interference fringe patterns found when using a nonflat object on the scanner cover: the left pattern was obtained with three fingertips and the right pattern when a cellotape roll was used.

D. Color of the fringes

The scanner manufacturer claims that the scanner lamp provides white light. Although our visual perception of the lamp is not white[8] but bluish, it is completely adequate to perform scans of color objects.

Should we expect to see all the colors of the visible spectrum in the interference pattern? If we set "image type: color" on the scanner software provided by the manufacturer, we found that there are only three main pastel colors in the image: blue, cyan, and purple.

We produce Fizeau fringes in an air layer of uneven thickness by illuminating it with a source that produces a wide range of wavelengths. However, the spectral power distribution of the scanner lamp (Fig. 3) is not flat at all. In fact the fluorescent scanner lamp has characteristic peaks, centered at 436, 488, 544, 584, and 612 nm, with different relative intensities. Furthermore, the phase difference between two consecutive waves (Fig. 5), assuming normal incidence, is given by Eq. (5), which depends very much on the wavelength. Moreover, we have a very low visibility pattern, which implies that the destructive interference condition does not mean zero intensity. Thus the color fringes are caused by the additive mixture of the different wavelengths of the lamp spectrum, with different weights depending on the interference intensity. However, those points of the air layer of similar thickness will display the same color band and the color pattern will be repeated when the thickness changes by half a wavelength, that is, the same phase difference in Eq. (5).

Those students who are able to handle mathematical software packages can write several lines of a code to simulate the color of the fringes, taking into account the lamp spectrum, the interference conditions, the additive mixture of the different wavelengths, and some basic concepts of colorimetry. We consider this final task to represent a satisfactory conclusion to the proposed experiment.

IV. SUMMARY

Class discussion is always beneficial for the students, but more so if the debate originates among them. We have discussed an experiment to produce Fizeau fringes at the small air layer in a flatbed scanner. Students can easily perform the first part of the experiment on their own. An interference pattern on the scanned image should puzzle them, but if the teacher points them in the right direction, they should gain considerable practical insight into interference, coherence, and colorimetry. Because the experiment is inexpensive and easy to do, it has become a useful complement to classical interference experiments in our laboratory and has proved its worth with the students. Variations of it are possible and the experiment can be adapted to the students' backgrounds and laboratory resources.

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APPENDIX

Temporal coherence is a measure of the ability of two relatively delayed light beams to form fringes. The coherence time we discussed in Sec. III B is useful when the light is quasimonochromatic and when its spectrum has a single, reasonably well-defined peak (as with a Gaussian spectral distribution). But when the spectrum has several peaks (as is the case with a scanner lamp), it is more difficult to provide a single useful definition for the coherence time. There are a multitude of definitions[2,9,10] involving the complex degree of coherence $\gamma(\tau)$. Our choice is the most common one. We define the coherence time τ_C by:

$$\tau_C=\int_{-\infty}^{\infty}|\gamma(\tau)|^2 d\tau. \tag{8}$$

For a Gaussian distribution Eq. (8) gives a coherence time of

$$\tau_C=\frac{0.664}{\Delta\upsilon}, \tag{9}$$

which differs slightly from Eq. (1). From a simple inverse Fourier transformation of the spectral power density of the source (in frequency units), we can obtain the associated complex degree of coherence $\gamma(\tau)$, using Wiener–Khintchine's theorem.[11,12]

We encourage the students to obtain the lamp spectrum in frequency units. Some of them will be tempted just to make a simple change in units from wavelength to frequency units. Converting one to the other, however, is not simply a matter of making the substitution $\nu=c/\lambda$ because the spectrum function, E_λ, is a distribution function and is defined differentially, so that[13]

$$E_\lambda d\lambda=E_\upsilon d\upsilon=E_\upsilon\frac{c}{\lambda^2}d\lambda. \tag{10}$$

Students can use a math package to obtain the coherence time (or equivalently the coherence length) by following the calculations in this Appendix. For the scanner lamp we obtained a value of 1870 nm (of comparable magnitude to the rough estimation made in Sec. III B), which is sufficient for the thin air layer to produce interference.

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[1]We used a HP ScanJet 6200 C for our experiments, but we have checked and found that Fizeau fringes can be obtained equally well with other commercial scanners with glossy white covers.
[2]M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1999), Chap. 10, pp. 554–619.
[3]M. V. Klein and T. E. Furtak, *Optics* (Wiley, New York, 1986), Chap. 5, pp. 263–307.
[4]F. L. Pedrotti, S. J. Pedrotti, and L. S. Pedrotti, *Introduction to Optics* (Prentice–Hall Intl., London, 1993), Chap. 10, pp. 211–221.
[5]Hewlett–Packard web site: http://www.hp.com.
[6]PR-650 spectroradiometer from Photo Research, Inc., 9731 Topanga Canyon Place, Chatsworth, CA 91311.
[7]CC-2022 spectrophotometer from Minolta Corporation, 101 Williams Drive, Ramsey, NJ 07446.
[8]According to G. Wyszecki and W. S. Stiles, *Color Science: Concepts and Methods, Quantitative Data and Formulae* (Wiley, New York, 1982), Chap. 6, p. 506, white is an ''attribute of a visual sensation according to which a given stimulus appears to be void of any hue and grayness.'' In our case the chromaticity coordinates, *x* and *y*, as measured by the spectroradiometer are 0.3072, 0.2897 with a correlated color temperature of 7380 K.
[9]J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985), Chap. 5, pp. 157–236.
[10]L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U.P., New York, 1995), Chap. 4, pp. 147–180.
[11]See Ref. 9, Chap. 3, pp. 73–79.
[12]See Ref. 10, Chap. 2, pp. 59–65.
[13]B. H. Soffer and D. K. Lynch, ''Some paradoxes, errors, and resolutions concerning the spectral optimization of human vision,'' Am. J. Phys. **67**, 946–953 (1999).