Calculating correlated color temperatures across the entire gamut of daylight and skylight chromaticities

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Natural outdoor illumination daily undergoes large changes in its correlated color temperature (CCT), yet existing equations for calculating CCT from chromaticity coordinates span only part of this range. To improve both the gamut and accuracy of these CCT calculations, we use chromaticities calculated from our measurements of nearly 7000 daylight and skylight spectra to test an equation that accurately maps CIE 1931 chromaticities x and y into CCT. We extend the work of McCamy [Color Res. Appl. **12,** 285–287 (1992)] by using a chromaticity epicenter for CCT and the inverse slope of the line that connects it to x and y. With two epicenters for different CCT ranges, our simple equation is accurate across wide chromaticity and CCT ranges (3000–10^6 K) spanned by daylight and skylight. © 1999 Optical Society of America

OCIS codes: 010.1290, 330.1710, 330.1730.

1. Introduction

A colorimetric landmark often included in the Commission Internationale de l'Eclairage (CIE) 1931 chromaticity diagram is the locus of chromaticity coordinates defined by blackbody radiators (see Fig. 1, inset). One can calculate this Planckian (or blackbody) locus by colorimetrically integrating the Planck function at many different temperatures, with each temperature specifying a unique pair of 1931 x, y chromaticity coordinates on the locus. (We follow convention and use chromaticity as a synonym for chromaticity coordinates.) Many natural and artificial light sources have spectral power distributions whose chromaticities either coincide with or are very near a particular chromaticity on the Planckian locus. Thus one can specify the color of such a light source simply by referring to its Planckian color temperature, which may differ significantly from its actual kinetic temperature.

Strictly speaking, if the chromaticity of a light source is off the Planckian locus, we must use the

term correlated color temperature (CCT) instead of color temperature to describe its appearance. Suppose that x_1, y_1 is the chromaticity of such an off-locus light source. By definition, the CCT of x_1, y_1 is the temperature of the Planckian radiator whose chromaticity is nearest to x_1, y_1. The colorimetric minimum-distance calculations that determine CCT must be done within the color space of the CIE 1960 uniformity chromaticity scale (UCS) diagram. However, we can use any kind of chromaticity diagram to plot measured or modeled chromaticities once we have calculated their CCT's.

In recent decades, researchers have repeatedly shown that both the chromaticity and the CCT of daylight provide good estimates of its visible-wavelength power spectrum.[1–7] Although the spectral irradiances E_λ of a given daylight phase uniquely determine its CCT, in principle metamerism means that CCT is not a good predictor of relative E_λ. In practice, however, atmospheric scattering usually acts as a fairly smooth spectral transfer function for sunlight, and this limits the metamerism of both daylight and skylight. We call these illuminants natural light collectively. Thus CCT is a useful shorthand for specifying the colorimetric and spectral characteristics of natural light. In fact the CIE describes the relative E_λ of typical daylight phases at the surface of the Earth by invoking CCT's ranging from 4000 to 25,000 K.[8] For example, Fig. 2 shows a measured spectrum of daylight E_λ for which the nearest Planckian chromaticity is that of a 5700-K blackbody (dashed curve). Thus this particular day-
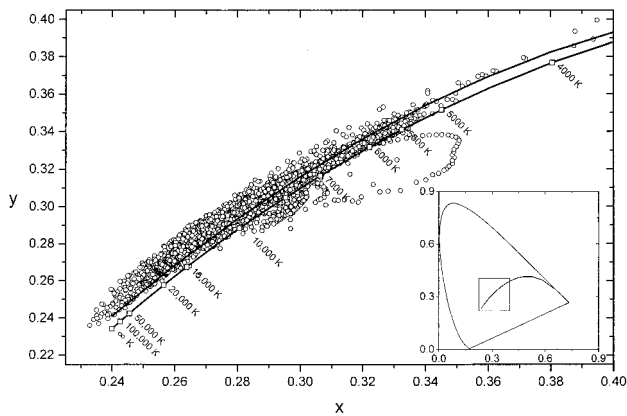
Fig. 1. CIE 1931 x, y chromaticities of our Granada, Spain, natural-light spectra (open circles) overlaid with the CIE daylight locus (solid curve) and Planckian locus (curve with open squares). The inset shows the entire CIE 1931 diagram and Planckian locus.

light spectrum has a chromaticity whose CCT is 5700 K.

Spectral measurements of natural light at many sites worldwide show that CCT varies between 3000 and $\sim 10^6$ K,[1–7] a range that must be covered by any simple equation that claims to calculate the CCT's of natural light accurately. Our goal here is to develop such an equation and to test it by using our measurements of nearly 7000 daylight and skylight chromaticities in Spain and the United States. Our measurements cover sky conditions ranging from clear to overcast and from midday to the dark limit of civil twilight; the altitudes of our observing sites range from near sea level to almost 3000 m. Given this wide variety of chromaticities, we can fairly assess the accuracy of our equation throughout the entire CCT range of natural light, including at high CCT's ($>25,000$ K) where earlier equations are either inaccurate or inapplicable.

In this paper we use target chromaticity to denote any off-locus chromaticity whose CCT we want to calculate. Clearly the more distant this target chro-
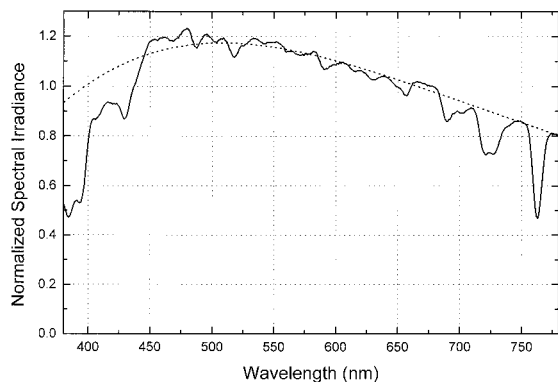

Fig. 2. Normalized spectral irradiances E_λ measured for particular daylight (solid curve) and calculated for a 5700-K blackbody (dashed curve). Because the 5700-K spectrum yields the Planckian chromaticity closest to the measured daylight chromaticity, this particular daylight has a CCT of 5700 K.

maticity is from the Planckian locus, the less accurately CCT can describe its color. Opinions vary on the maximum distance at which CCT is perceptually meaningful. However, we take the fairly conservative stance that, because our chromaticities have a mean 1960 u, v Euclidean distance of only 0.00422 from the Planckian locus (standard deviation, 0.0026), CCT is a valid measure of the color of natural light.

2. Existing Methods for Calculating Correlated Color Temperature

During the past four decades a variety of algorithms and equations have been proposed for calculating CCT across different ranges of color temperature. Originally CCT was calculated by computing the 1931 x, y chromaticities (or 1960 u, v) for a light and then consulting charts developed by Kelly.[9] Within a chart a CCT was found by interpolation between two isotemperature lines (lines of constant CCT) closely flanking the target chromaticity. Nowadays faster and more accurate numerical interpolation is used instead of Kelly's graphic interpolation. In 1968 Robertson proposed a CCT algorithm that has since been widely adopted.[10] His algorithm uses a fairly simple technique for numerical interpolation between isotemperature lines such as Kelly's. For his highest-resolution work, Robertson chose 31 isotemperature lines and tested the accuracy of his algorithm versus the known CCT's of 1800 chromaticities. His errors were quite small, reaching a maximum of 5.4 K in the 2000–14,000-K range and 450 K in the 50,000–10^5-K range.

Since the Kelly and Robertson research of the 1960's, other researchers have developed numerous methods for calculating CCT; here we outline some of those that are relevant to determining CCT's of natural light. In 1978 Schanda et al.[11] used a simple binary search algorithm to generate two seventh-order polynomial equations that map the Planckian temperature onto its corresponding x, y chromaticities in the 2000–50,000-K range. In earlier research Schanda and Dányi[12] used a similar Planckian-locus fitting to calculate the CCT's of daylight chromaticities. Krystek[13] in 1985 suggested a CCT algorithm based on a rational Chebyshev approximation to the Planckian locus, combined with a bisection procedure. Krystek's errors in determining CCT were $<0.03\%$ for a CCT of 1000 K and 0.48% for a CCT of 15,000 K. As Krystek noted, these errors are not too significant given that uncertainties in measuring chromaticities are normally 10 times greater, reaching 5% at a CCT of 15,000 K. Two years later, Xingzhong[14] developed empirical equations to calculate CCT from 1960 chromaticities u, v in the range of 1666–25,000 K, obtaining errors comparable with Robertson's. Xingzhong's fifth-order polynomials are functions of two variables. The first variable is the angle formed by three chromaticities: (1) a target u, v; (2) a u, v below the Planckian locus where the isotemperature lines in some CCT range (almost) converge; and (3) a u, v to the right of (2) and

at the same v. [McCamy calls a convergence point such as point (2) an epicenter of convergence.[15,16]] Xingzhong's second variable is the Euclidean distance between an epicenter and the target u, v. Although Xingzhong's algorithm is simpler than Robertson's, in its present form it covers a much smaller CCT range than his and it remains more complicated than the one that we develop below.

More recently, McCamy proposed a third-order polynomial equation for computing CCT from CIE 1931 x, y. As his predecessors did, McCamy noted that "the isotemperature lines for CCTs of princip[al] interest nearly converge toward a point on the [CIE 1931] chromaticity diagram," adding that his function depends on "the reciprocal of the slope of the [isotemperature] line from that point to the chromaticity of the light."[15] Because there is no single convergence point (or epicenter) for a broad CCT range, McCamy used intersections of 16 pairs of CCT isotemperature lines between 2222 and 12,500 K to calculate an epicenter that yielded the smallest CCT errors in that range. His resulting best-fit CCT epicenter is at $x_e = 0.3320$, $y_e = 0.1858$.

McCamy's polynomial formula for CCT is

$$\text{CCT} = an^3 + bn^2 + cn + d \qquad (1)$$

with inverse line slope n given by

$$n = (x - x_e)/(y - y_e) \qquad (2)$$

and constants a, b, c, and d. Yet, as our daylight and skylight spectra show, the CCT range for which McCamy's equation gives good results (2000–12,500 K) is much smaller than that of natural light (3000 to $\sim 10^6$ K).

3. Can We Calculate the True Correlated Color Temperature of a Chromaticity?

Not surprisingly, the numerous CCT algorithms described above often yield different results for the same chromaticity coordinates. Although these algorithms all start with the same CCT definition, their CCT's differ because their assumptions and computational methods differ. Depending on its implementation, even the same algorithm can produce different CCT's. For example, we measured our daylight and skylight spectra with two spectroradiometers[17,18] that employ the Robertson algorithm; yet at high CCT's ($>10^5$ K) we found discrepancies of 2% at identical chromaticities. Furthermore, although the Li-Cor spectroradiometer has a maximum CCT of 10^5 K (higher-temperature values are simply called out of range), the Photo Research instrument calculates CCT's as high as $\sim 9 \times 10^5$ K.

To assess the accuracy of any simple CCT algorithm, clearly we must avoid such discrepancies and calculate what we call reference CCT's as accurately as possible. To do so, we start with the definition of CCT: The CCT of any CIE 1960 target chromaticity u_1, v_1 is the color temperature that yields the minimum Euclidean distance between u_1, v_1 and the Planckian chromaticity u_P, v_P of the color tempera-

ture. We call this distance $\Delta(u, v)$. Our algorithm for calculating reference CCT's uses a simple binary search across color temperature that minimizes $\Delta(u, v)$. Because our reference CCT's are calculated directly from the definition of CCT, they are as close as one can get to the true CCT of a chromaticity, subject to the uncertainties noted below.

Two of us (Hernández-Andrés and Lee) independently wrote algorithms for calculating reference CCT's. A step-by-step outline of these algorithms follows:

(1) A target CIE 1931 x_1, y_1 is first converted to its corresponding CIE 1960 u_1, v_1.

(2) We choose three initial trial CCT's of 10^2 and 10^{10} K and their logarithmic mean (10^6 K).

(3) For each of the three trial CCT's we calculate a blackbody E_λ spectrum at 4-nm intervals between 380 and 780 nm.

(4) Following colorimetric convention, we use Riemann sums to approximate the colorimetric integrals of each blackbody spectrum. From the resulting tristimulus values, we calculate three different pairs of CIE 1960 u_P, v_P, one pair for each trial CCT.

(5) We then calculate $\Delta(u, v)$ for each u_P, v_P and identify the minimum distance $\Delta(u, v)_{\min}$. The CCT that produces $\Delta(u, v)_{\min}$ is at the center of our next round of CCT searches.

(6) With each new search round we reduce the search range by a fairly cautious convergence factor of 0.7. Our experience shows that using a smaller convergence factor can cause the miscalculation of some high-temperature CCT's. In step (2) our initial trial CCT's differed by factors of 10^4. In the second search round, each trial CCT differs by a factor of 7000 from the next one; in the third search round, trial CCT's differ by a factor of 4900. Thus, if the first $\Delta(u, v)_{\min}$ came from a trial CCT of 10^6 K, the second search round would use trial CCT's of 142.857, 10^6, and 7×10^9 K. After three new trial CCT's are calculated the algorithm loops back to step (3). Because we have already calculated a blackbody E_λ spectrum for one of the new trial CCT's, we do not need to recalculate it, thus increasing the search speed.

(7) When the maximum and the minimum trial CCT's differ by less than 0.1 K, we have identified the reference CCT for x_1, y_1 with sufficient accuracy. We exit the search loop and return as our reference CCT, the middle CCT from the last search round, rounded to the nearest kelvin.

Although such binary search algorithms have the virtue of being fairly simple to write, they have the vice of being very slow. In our experience the binary search algorithm outlined above can be as many as 10^4 times slower than the simple equation that we develop below. Such a slow performance might not matter on a fast desktop computer, but the slow speed of the binary search and its greater complexity *are* problematic if computing speed is at all limited (e.g., in a field-portable spectroradiometer). Occam's ra-
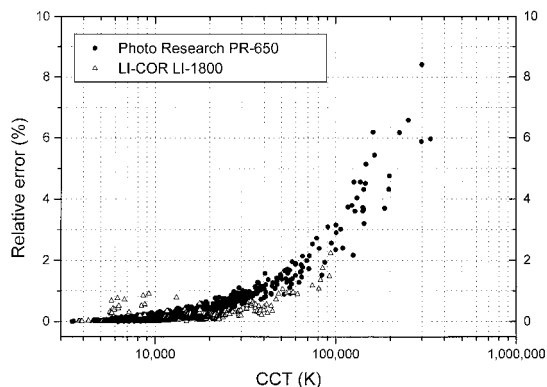
Fig. 3. Relative errors of the Robertson algorithm[10] CCT's calculated by our spectroradiometers compared with reference CCT's from a binary search algorithm that is given the same CIE 1931 x, y chromaticities.

zor ultimately means that simpler, faster solutions to any complicated problem are at least worth studying.

Note that even our binary search routines can disagree slightly, depending on, for example, the initial range of CCT's searched, our chosen rate of convergence, the floating-point arithmetic used by a programming language, and especially how we colorimetrically integrate the Planck function. Nevertheless the agreement between our binary search algorithms is quite good, with an average CCT difference of only 0.958%. The algorithms differ by $\leq 0.143\%$ for CCT's of $\leq 37,543$ K and have a maximum difference of 5.1% at a CCT of $\sim 9 \times 10^5$ K. As these results suggest, calculating CCT is ultimately a slightly indeterminate exercise. In other words, depending on the computational factors listed above, even a single binary search algorithm may calculate slightly different reference CCT's for a given target chromaticity. Thus we are left with an irreducible ambiguity in calculating CCT. Fortunately, this ambiguity is mostly irrelevant in colorimetric terms—all our algorithms, whether a binary search or simple equation, only rarely produce CCT differences that are colorimetrically perceptible. In other words, when our different CCT algorithms start with

the same target chromaticity, only rarely are the resulting Planckian chromaticities >1 just-noticeable-difference[8] (JND) apart on the CIE diagram.

Given the irreducible ambiguity of any CCT calculations, we arbitrarily settle on using Hernández-Andrés's binary search algorithm for our reference CCT's. In the interest of CCT consistency (and of being as accurate as possible), we use this algorithm to calculate reference CCT's across the wide gamut of natural-light chromaticities. With reference CCT's in hand we can now assess the accuracy of CCT's calculated by our spectroradiometers. Figure 3 shows the relative errors of CCT's calculated by software on the two instruments compared with reference CCT's from our binary search algorithm, independent of any measurement errors. For CCT >80,000 K, the instrument CCT errors exceed $\sim 2.5\%$, reaching a maximum error of 8.4% at $\sim 3 \times 10^5$ K. In the end we reduced our maximum calculated CCT slightly from 10^6 to 8×10^5 K, noting (1) that at higher temperatures, physically realizable graybody radiators are moot for colorimetry (i.e., trying to create the radiant equivalent of such bluish daylight or skylight is impractical) and (2) that the 1931 x, y distance between 8×10^5 and ∞ K is 6×10^{-4}. This distance is comparable with the colorimetric uncertainty of our spectroradiometers, especially at the low illuminance levels at which the highest CCT's occur.

4. Our Measurements of Some Natural-Light Spectra

The majority of our natural-light measurements, 5315 daylight and skylight spectra, were taken in Granada, Spain. Our observation site there was the flat roof of the Sciences Faculty at the University of Granada, which is situated in a nonindustrial area of Granada. The data were compiled over three years and measured with a Li-Cor LI-1800 spectroradiometer.[17] Table 1 lists other pertinent details about our sites.

Our Granada data consist of (a) 2600 daylight measurements (global E_λ on a horizontal surface in all weather), (b) 1567 clear skylight measurements [spectral radiance taken with a 3° field-of-view tele-

Table 1. Temporal and Spatial Details of Our Seven Spanish and U.S. Observing Sites, Listed in Decreasing Order of the Number of Spectra Measured at Each Site

Site Name	Latitude (°N)	Longitude (°W)	Altitude (m)	Site Type[a]	Measurement Dates
Granada, Spain	37.183	3.583	680	U	February 1996– November 1998
Owings, Md.	38.688	76.582	15	R	6 January– 24 October 1998
Annapolis, Md.	38.984	76.484	27	U	10 September– 11 November 1998
Marion Center, Pa.	40.810	79.080	451	R	5–6 September 1998
Perkasie, Pa.	40.417	75.339	153	R	19 December 1997
Cheektowaga, N.Y.	42.909	78.750	215	U	2 January 1998
Copper Mountain, Colo.	39.486	106.161	2960	R	13 March 1998

[a]Rural sites are denoted R and urban sites U; all non-Granada sites are in the United States.
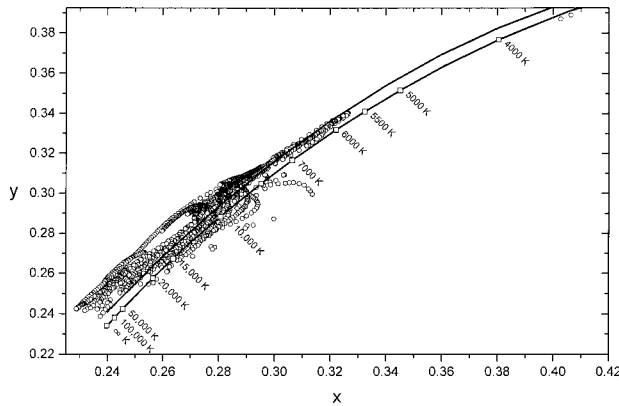
Fig. 4. CIE 1931 x, y chromaticities of our U.S. natural-light spectra (open circles) overlaid with the CIE daylight locus (plain curve) and Planckian locus (curve with open squares).

Table 2. CIE 1931 Best-Fit Colorimetric Epicenters x_e, y_e and Constants for Eq. (3)

Constants	Valid CCT Range (K)	
	3000–50,000	50,000–8 $\times 10^5$
x_e	0.3366	0.3356
y_e	0.1735	0.1691
A_0	−949.86315	36284.48953
A_1	6253.80338	0.00228
t_1	0.92159	0.07861
A_2	28.70599	5.4535 $\times 10^{-36}$
t_2	0.20039	0.01543
A_3	0.00004	
t_3	0.07125	

Note: Equation (3) has only two exponential terms in the higher CCT range.

scope at different points in the sky hemisphere], and (c) 1148 twilight measurements (global E_λ on a horizontal surface for clear and overcast skies). Consistent with the precision of our spectroradiometers, we list only four significant figures for CCT's here, although we used more in our calculations. Two Granada skylight measurements do not have an associated CCT because their chromaticities lie too far from the Planckian locus. Figure 1 shows the CIE 1931 chromaticities of all 5315 Granada measurements overlaid with both the Planckian locus and the CIE daylight locus.

Our 1685 U.S. spectra are also dominated by horizontal E_λ with most of them acquired just before sunset (unrefracted solar elevation, $h_0 < 5°$) or during civil twilight ($h_0 \geq -6°$). Clear skies produced 1321 of these spectra, 339 are from overcasts, and 25 are from partly cloudy skies. The U.S. observation sites alternate between rural and urban, and most are at fairly low altitudes in the eastern United States. Figure 4 shows the CIE 1931 chromaticities of our U.S. natural-light spectra, and these chromaticities span a CCT gamut quite similar to that measured at Granada.

We used our U.S. and Spanish spectra to calculate chromaticities that test the CCT equation that we propose below. Collectively, our measured spectra yield an unprecedentedly broad gamut of natural-light chromaticities and CCT's. This gamut is broad in part because our data include chromaticities from many clear and overcast twilights, and high-temperature (i.e., bluish) CCT's are most often found during the darker half of civil twilight. Only by having this wide a variety of natural-light chromaticities in hand can we be confident of the generality and accuracy of our CCT equation.

If we use McCamy's equation [Eq. (1)] to calculate CCT, relative errors for our Granada data exceed 5% for CCT's of >14,000 K and increase to 70% at high CCT's. Although McCamy correctly notes that his equation "is entirely adequate for all practical purposes in the range [of 2000–12,500 K]",[15] because much of our data (19.8%) lies outside this range, Eq.

(1) is inadequate for our purposes, i.e., quickly and accurately calculating CCT across the entire gamut of natural-light chromaticities.

5. Our Proposed Correlated Color Temperature Equation

Although McCamy's approach has considerable promise, we need to extend its useful CCT range. Using the same parameter n as McCamy, we offer a simple alternative to Eq. (1):

$$\text{CCT} = A_0 + A_1 \exp(-n/t_1) + A_2 \exp(-n/t_2)$$
$$+ A_3 \exp(-n/t_3) \quad (3)$$

with constants A_i and t_i. As McCamy did, we began by using only one colorimetric epicenter in Eq. (3). However, this simplicity came at the price of unacceptably large errors at high CCT's. Although CCT errors were tolerably small below 70,000 K (<5%), they became as large as 53% above 10^5 K. Because CCT accuracy depends critically on our choice of epicenter, we followed Xingzhong's lead[14] and added a second epicenter at high CCT's. Changing epicenters requires changing the Eq. (3) constants A_i and t_i, and we list these constants in Table 2. For the CCT range of 3000–50,000 K we use an epicenter at $x_1 = 0.3366$, $y_1 = 0.1735$, but for CCT > 50,000 K we shift it slightly toward the origin ($x_2 = 0.3356$, $y_2 = 0.1691$). Note that because the Planckian locus is a broad concave-down curve, the epicenter of a curve segment that fits any part of the locus will be located far below it. Figure 5 shows the good agreement between reference CCT's and those from Eq. (3) for all our measured chromaticities.

Note that CCT in Eq. (3) increases only slightly (<2%) when we change epicenters and constants at 50,000 K, although n itself shifts significantly in Eq. (3). A simple way to pick the correct constants for a given x, y is to first assume that CCT ≤ 50,000 K and use values from the middle column of Table 2. If this choice yields CCT 50,000 K, recalculate Eq. (3) by using the right column of Table 2.

Our error analysis of the two-epicenter form of Eq. (3) appears in Tables 3 and 4. For all 7000 of our
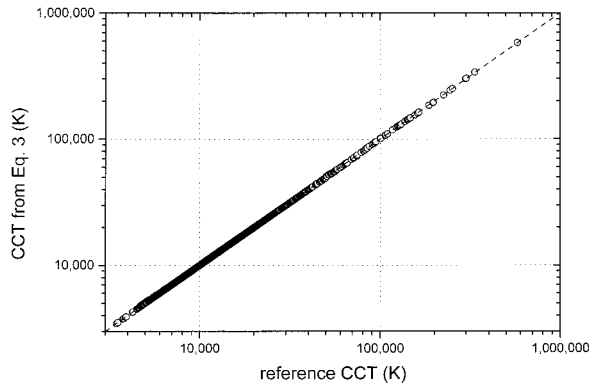
Fig. 5. Equation (3) CCT's compared with reference CCT's calculated for the same chromaticities. If Eq. (3) were exact, all points would lie exactly on the main diagonal (dashed line).
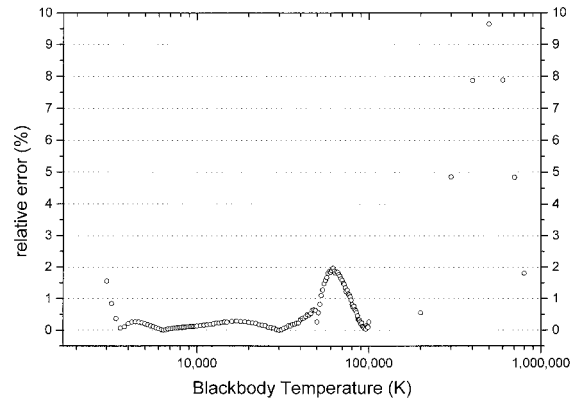

Fig. 6. Relative CCT errors for Eq. (3) when it is given the CIE 1931 x, y chromaticities of 138 Planckian radiators of known temperatures. Note that the high-temperature Planckian chromaticities lie far from the locus of natural-light chromaticities used to develop Eq. (3) (see Figs. 1 and 4). Thus, when Eq. (3) is properly limited to calculating CCT's for natural-light chromaticities, its errors will be much smaller (see Table 4) than those shown here.

daylight and skylight spectra the maximum relative CCT error in Eq. (3) is <1.74% over the entire CCT range ($3000 – 8 \times 10^5$ K) and its mean error is 0.084%. Table 4 shows the maximum and mean relative errors in each CCT range for Eq. (3) as well as the maximum and the mean CIE 1931 Euclidean distances (i.e., colorimetric errors) between its CCT's and corresponding reference CCT's.

Not surprisingly, the largest colorimetric errors occur at low CCT's where a small change in CCT causes rather large changes in chromaticity. The maximum relative error of Eq. (3) is <1.74% for a CCT of 4792 K. Although this relative error results from an absolute CCT error of only ~83 K, it does yield a

Table 3. Percentile Distribution of Eq. (3) CCT Errors Compared with Reference CCT's

Relative CCT Error (%)	Number of CCT's	Percentile (%)
<0.1	5442	77.8
<0.2	6491	92.8
<0.5	6905	98.7
<1.0	6971	99.6
<2.0	6998	100

Note: A binary search algorithm calculates reference CCT's from our measured daylight and skylight chromaticities.

possibly suprathreshold colorimetric error of 0.00317. Conversely, at large CCT's a clearly subthreshold chromaticity change can cause a large shift in CCT. Yet these results are not contradictory if we remember the fundamental nature of CCT—it maps a spectral function that is exponential in inverse temperature (the Planck function) onto a color space in which we usually use only simple additive or multiplicative transfer functions (e.g., the convolution of E_λ and the spectral reflectance of a surface). Thus the relationship between CCT and chromaticity distances is neither direct, linear, nor familiar.

If chromaticities are far from the natural-light locus defined by the cluster of dots in Figs. 1 and 4, the CCT errors for Eq. (3) may be larger. To test its accuracy in these conditions, we calculate the chromaticities of 138 Planckian radiators whose color temperatures (CT's) range from 3000 to 8×10^5 K. Given these chromaticities Fig. 6 shows that the two-epicenter form of Eq. (3) reproduces the corresponding CT's with acceptable accuracy (relative errors, <5%) across the $3000–3 \times 10^5$-K range. Yet at 5×10^5 K, Eq. (3) errors increase to 10% because the locus

Table 4. Maximum and Mean Percentage CCT Errors from Eq. (3) and Its Maximum and Mean CIE 1931 Colorimetric Errors Compared with Reference CCT's for Our Measured Daylight and Skylight Spectra

CCT Range (K)	Relative Error		CIE 1931 Colorimetric Error		CCT's in Range
	Maximum	Mean	Maximum	Mean	
3000–5000	1.73	0.71	0.00317	0.00131	39
5000–9000	1.53	0.06	0.00268	0.00007	3726
9000–17,000	0.62	0.09	0.00041	0.00006	2632
17,000–50,000	1.02	0.14	0.00020	0.00004	520
$50,000–10^5$	1.37	0.56	0.00012	0.00005	46
$10^5–8 \times 10^5$	1.40	0.58	0.00007	0.00002	35

Note: By comparison, MacAdam 1931 x, y color matching ellipses on the Planckian locus from 3000 to 8×10^5 K have a mean semimajor axis of 0.0029294 (standard deviation, 2.2424×10^{-4}) and a mean semiminor axis of 9.3694×10^{-4} (standard deviation, 1.6345×10^{-4}).

of its underlying data is so far from the Planckian locus at a high CT (errors do decrease at higher CT's). Quantitatively, when the CIE 1931 x, y colorimetric distance between the natural-light locus and a target chromaticity is ~0.01 (e.g., the high-temperature Planckian chromaticities in Figs. 1 and 4), the CCT errors of Eq. (3) *may* be significantly larger than expected. Mathematically this occurs because accuracy at high CCT's is so sensitive to our choice of epicenter and constants in Eq. (3). Perceptually, however, the data in Table 4 suggest that the associated colorimetric errors are likely to be subthreshold (i.e., <1 JND) at these high CCT's. If target chromaticities are too far from the Planckian locus, the idea of CCT itself becomes perceptually meaningless.

6. Conclusions

As noted above, CCT can be calculated from CIE 1931 chromaticity coordinates x and y in many different ways. Complicated algorithms and simple equations alike have been proposed and used for several decades. Although we can always calculate reference CCT's by using a binary search algorithm, this computationally intensive approach is not always desirable. Underlying all of this variety is the common problem that CCT algorithms disagree numerically (if not perceptually) at high CCT's. In reexamining this problem, we chose a middle path between reference-level accuracy and McCamy's simplicity by using a two-epicenter exponential equation for calculating CCT [Eq. (3)]. The result is that our CCT's are nearly as accurate as those calculated by the most complex algorithms, even for chromaticities fairly far from the natural-light locus.

Although our CCT equation is not infallible, we believe that it strikes a good balance between computational simplicity and colorimetric accuracy. Equally important, our work indicates that Eq. (3) is both quite accurate and useful for most practical colorimetry. Our many measurements of daylight and skylight spectra convince us that Eq. (3) and Table 2 can be used confidently over the entire gamut of natural outdoor lighting seen at most locations, regardless of weather or time of day.

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