Linear Bases for Spectral Reflectance Functions of Acrylic Paints

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Abstract: A linear model for representing reflectances has been developed from a group of 5574 samples of acrylic paint on paper. Using acrylic paints makes easy the generation of a large variety of samples by mixing, due to the high miscibility among these kinds of pigments: this point was the key to achieve a great spatial homogeneity in our samples. Besides, these kinds of paints keep their chromatic properties stable over time. The first 7 vectors of the so-called overall linear basis were sufficient for a more than adequate mathematical representation of the spectral-reflectance curves. A study by hue groups of the mathematical properties of these curves indicates that the use of a hue basis of representation implies, on the average, a reduction in 1 or 2 of the number of vectors needed in order to achieve results analogous to those of the overall basis. © *1998 John Wiley & Sons, Inc. Col Res Appl, 23, 39–45, 1998*

Key words: linear model; reflectance; acrylic paints

INTRODUCTION

The visual information that we receive from nontransparent objects surrounding us comes from diffuse or specular reflected light from the objects themselves and from the light reflected from the other objects situated in that setting. The spectral distribution of these lights depends on the reflectance properties of the surfaces of the objects and on the spectral distribution of the illuminant. In-depth knowledge of these properties is unquestionably useful in many technical and scientific applications in which the aim is to capture and process the optical information.

Thus, for example, in the field of Artificial Vision, problems such as the recognition and identification of color of the objects or the classification of the materials according to their nature (metals, organic materials, plastics, etc.) are intimately linked to the knowledge of the interaction of light with the material generating the diffuse and/or specular reflection. In the same sense, an immediate application of the study of the spectral reflectance of the diffuser object is the graphic simulation of that object by the computer.

The study of properties associated with light reaching us from the objects and the possible processing of this information have the aim of explaining the properties of the materials and their appropriate use or simulation. In so doing, we can develop adequate systems of representing reflectance. The search for ways of representing reflectance more effective than the canonical way[1–8] has been a constant characteristic in developing computational algorithms to recognize and identify color in the sphere of Artificial Vision. From the beginning, there was a need to find bases that make use of the degree of underlying correlation in the spectral-reflectance functions recognized in nature. The canonical representation, despite being more natural, had little virtue aside from the direct identification of the physical characteristics of the functions being developed.

In the literature related to this matter,[1–7] various works demonstrate the phenomenon observed on studying broad groups of spectral reflectances: despite moving us initially in an isomorphic Euclidean space to $\mathbb{R}^n$, the reflectance groups appear to be limited rather precisely to subspaces

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of $\mathbb{R}^n$ that are notably smaller in dimension than the total space. This physical fact motivates the search and generation of appropriate bases of reflectances.

Thus, in principle, it is vital to have a good choice of the group of reflectance curves. That is, if we want to generate a basis as operative as possible, then the initial group of curves to be studied should be sufficiently broad and varied to be as representative as possible of the greatest range of curves of the objects that surround us. This group can be formed by curves of natural samples (leaves, soils, fruit, etc.) or artificial (tiles, colored fabrics, acrylic paints, etc.), since, as long as no fluorescence or phosphorescence occurs, the spectral-reflectance curves of natural samples are indistinguishable from those of artificial objects, sharing the same characteristics of smoothness, width and specific shape.[3,7]

It is useful to remember that, depending on the sphere in which one works, either color or reflectance recovery, the number of parameters and/or vectors of a linear model vary,[8] if our goal is the mathematical recovery of the reflectance functions themselves (for example, when we study the phenomenon of color constancy under different illuminants), the number of parameters needed will rarely fall below 6. If, on the other hand, we limit our aim to color recovery (assuming that we always carry out the recovery under the same illuminant or with color synthesis by computer,[9] for example), the number of parameters would be notably reduced (4 vectors, or at times 3, can be sufficient, depending on the tolerance levels in the recovery). We should remember that the human visual system lacks the capacity of analysis—the perception of color occurs after the tristimulus integration of the spectral stimulus, in the process losing all specific information relative to spectral aspects of this stimulus. This is the reason for a reduction of the number of parameters necessary in the linear model: the recovery of the color is less demanding than the recovery of the spectral curve.

In light of topics reviewed above regarding the study of reflectances, our objective is the generation of an adequate linear basis for the representation of reflectances. Bearing in mind this point, we achieve our study on a large set of spectral reflectance curves obtained from commercial acrylic paint samples on paper and clustered by a hue classification, in order to study their properties independently. We chose these kinds of paints because they are very common in art and industry due to their chromatic properties and availability, avoiding the use of sophisticated materials and techniques for generating the sample collection.

METHOD

As mentioned above, it is fundamental to have available a gamut of spectral reflectance curves as broad and varied as possible in order to cover the greatest range of observed variation in the functions of opaque objects that surround us. We decided that our collection of samples should satisfy a series of basic prerequisites: they should represent as faithfully as possible the properties of the curves of the objects found both in nature[6,10] and in the objects and materials of daily use,[8,11,12] in terms of smoothness of shape. The necessity of this condition is clear: the wider the set in representing shaping properties, the more suitable in representing spectral reflectance functions, even though they have not taken part in the original set. In addition to a great spatial homogeneity, each sample should maintain its chromatic properties stable over time.

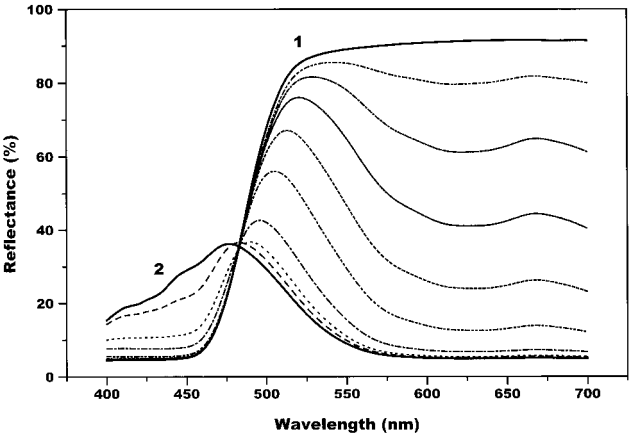

FIG. 1. Spectral reflectance curves of some samples generated from two specific basic colors: yellow (solid line #1) and blue (solid line #2).

Most of the works on this subject to date have used the samples of Krinov[3,6,10] or a selection of samples from the *Munsell Color Book*.[4,5,7] We chose to generate our own sample set using acrylic paint on paper, creating a total of 5,574 different samples, each 4×4 cm in area, with a homogeneous texture. Using acrylic paints made it easy to generate a large variety of samples by mixing due to the high miscibility among this kind of pigments: this point was the key to achieving a great spatial homogeneity in our samples. The samples of the entire group were classified into 5 hue categories: red, yellow, green, blue, and purple, with 1080, 1062, 1208, 1064, and 1160 samples each, respectively. Those samples let us not only study the characteristics of the functions in each separate hue group, but also obtain the best linear basis for a specific spectral reflectance curve when we know its hue. Let us imagine we are only interested in representing spectral reflectance functions of red samples; it would be better to use a specific red-hue basis instead of an overall one, because the number of vectors necessary could be reduced.

To generate these samples, we used 24 basic commercial acrylic colors for artists. Beginning from these 24 basic colors (each one defined from a determinated type of pigment) we obtained our samples by mixing 2 or 3 of these colors within a different ratio. Figure 1 shows the spectral reflectance curve of some samples generated from 2 specific basic colors, a yellow (solid curve 1) and a blue (solid curve 2) one. As we can observe, different

curves have been generated, gradually covering different hues from the original yellow to the original blue, passing through a gamut of greens. However, the curve obtained as the result of mixing specific basic colors is, from a mathematical viewpoint, not just a linear combination of the original reflectance curves, as we can see from Fig. 2. In this figure we can observe the result of expressing the obtained curve (dotted line) by mixing the two originals (1) and (2), and by generating the linear combination of these two originals (dashed line). If it were exactly the linear combination it would not be necessary to obtain 5,574 samples by mixing, since we could have achieved the principal value decomposition over the reflectance curves of the original 24 basic colors.

Each sample's spectral reflectance was measured with a HunterLab UltraScan™ spectrophotometer, controlled by a PC-computer using specific SpecWare V2.0™ software. Diffuse spectral reflectance was measured for every color sample. Samples were diffuse illuminated with simulated CIE D65 spectral distribution filtered to eliminate infrared specimen heating. The geometry of observation was 8° from specimen normal. The measuring range was from 400–700 nm with a 5-nm wavelength interval. From each reflectance $\rho(\lambda)$, we obtained the symmetric $R(\rho)$ after calculating the dyadic product of this reflectance in itself:

$$R_{ij}(\rho) = \rho(\lambda_i)\rho(\lambda_j) \quad \text{with: } 1 \le i, j \le 61. \qquad (1)$$

Thus, the matrix $R(\rho)$ saved the information associated with reflectance $\rho(\lambda)$. Adding all the matrices $R(\rho)$ constructed for all the samples of the same hue group, we obtained the Correlation Matrix, $R_T(\rho)$, of this group. After formulating this correlation matrix, we submitted it to a principal value decomposition process, obtaining its eigenvalues and eigenvectors.

Once we had obtained the eigenvalues and eigenvectors from each hue group, we arranged the latter in descending order of their eigenvalue—this criterion enabled us to determine the eigenvectors that would have a greater contribution in the construction of the basis. This independent study of each hue group enabled us to analyze separately the intrinsic characteristics comprising each group.

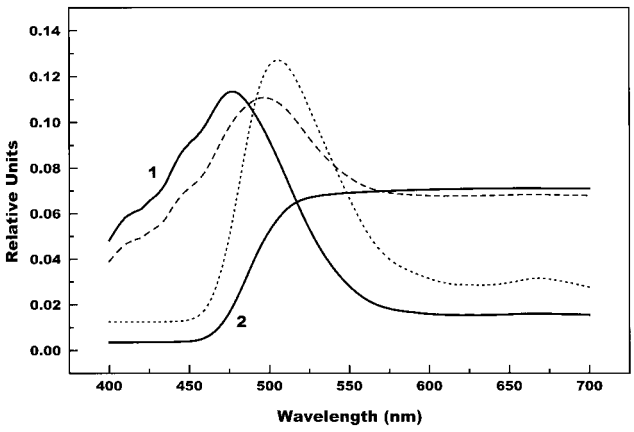

FIG. 2. Example of lack of linearity after mixing. The dotted line is the spectral reflectance curve obtained by mixing the two curves (solid lines #1 and #2) corresponding to two original basic colors. The dashed line is the corresponding linear combination of these two originals.

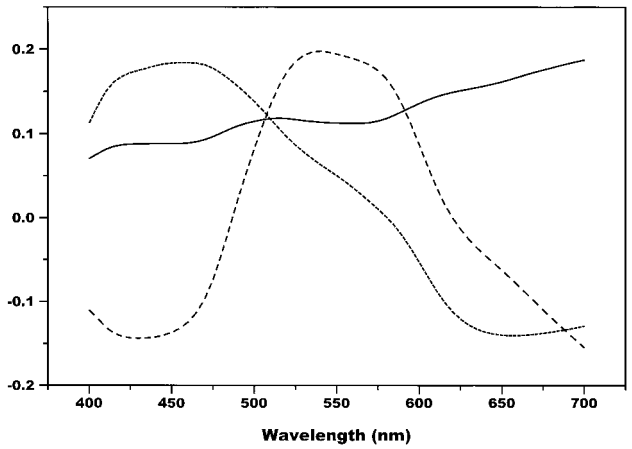

FIG. 3. Vectors #1 (solid line), #2 (dotted line), and #3 (dashed line) of the overall basis.

We followed the same process to obtain the overall basis from the sum of the correlation matrices of the 5 hue groups (Figs. 3 and 4 show the first 6 vectors of this basis)*.

Having recorded the collection of reflectances, we proceeded to adjust the hue bases and the overall basis, reproducing these according to the expression:

$$\rho_A = \sum_{i\,1}^{p} [\rho(\lambda)\cdot v_i(\lambda)]v_i(\lambda), \qquad (2)$$

where $\rho(\lambda)\cdot v_i(\lambda)$ is the usual scalar product between the reflectance and the ith vector of the basis, and p is the number of vectors with which we want to recover the function.

To be able to compare the goodness of the recovery, we define a goodness-fitting coefficient (GFC) according to the following expression:

$$\text{GFC} \equiv 1 - 2\,\frac{\sum_{j=1}^{61} |\rho(\lambda_j) - \rho_A(\lambda_j)|}{\sum_{j=1}^{61} |\rho(\lambda_j) + \rho_A(\lambda_j)|} \quad \text{with:}$$

$$\lambda_j = (395 + j5)\ \text{nm}, \qquad (3)$$

where $\rho_A(\lambda)$ is the curve recovered by a given model and number of vectors, and $\rho(\lambda)$ is the original spectral reflectance.

As reflected by the definition of the coefficient, the

* Anyone interested in our data set or bases can mail the corresponding author and will be provided the data on diskette or via e-mail.

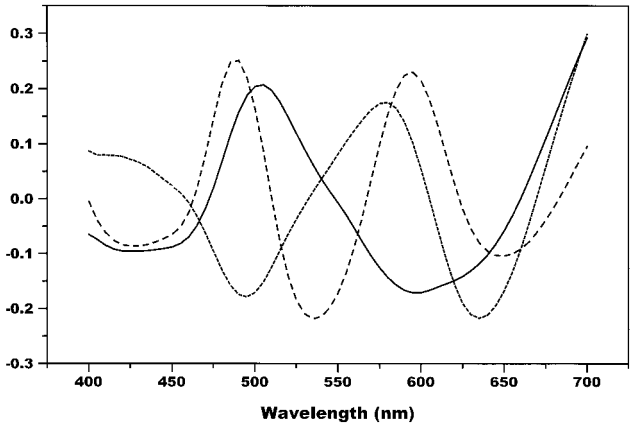

FIG. 4. Vectors #4 (solid line), #5 (dotted line), and #6 (dashed line) of the overall basis.

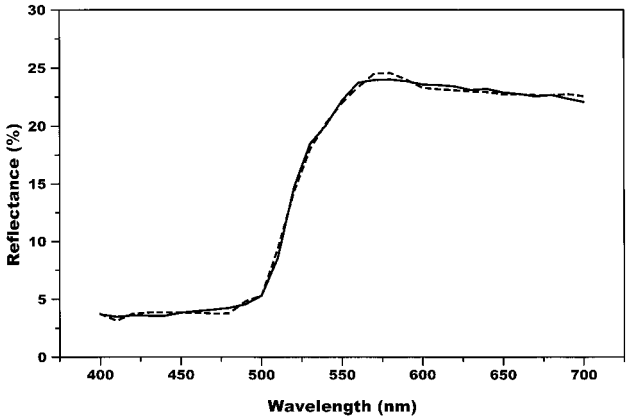

FIG. 6. Example of reconstruction (dashed line) of an original spectral reflectance function (solid line) obtaining a GFC = 0.9963.

closer the recovered curve resembles the original, the closer the value of this coefficient approaches unity. The definition of this coefficient is meant to give a direct average of the relative difference between the original and the recovery for each wavelength. One of the advantages of the simplicity in its definition is that it is easily identifiable with the relative error introduced upon taking the recovered curve instead of the original as the spectral-reflectance function. On the other hand, as much as it might seem trivial, this definition contributes to the symmetric character desirable in this type of coefficient: it provides the same value when comparing the recovery and original as when comparing the original with the recovery.

In practical terms, this simplicity in its definition enables us easily to translate specific values of the coefficient to quality in the fit. Thus, on the average, from a merely functional perspective, we consider a recovery good when it reaches a GFC of 0.925, very good when it reaches 0.950, and excellent when it reaches or exceeds 0.990. Figures 5 and 6 present two examples of the graphic meaning of the different values in the GFC (0.9377 and 0.9963 for Figs. 5 and 6, respectively).

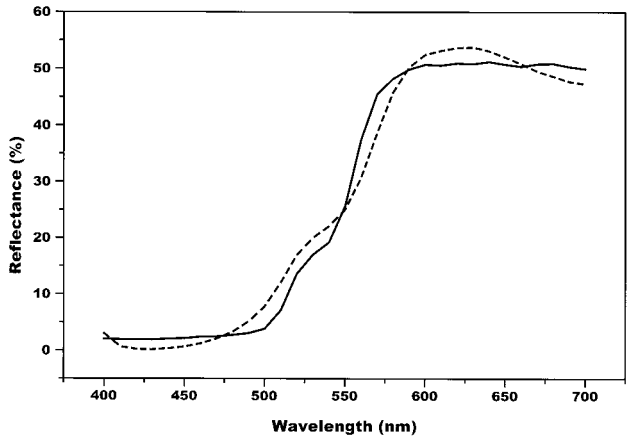

FIG. 5. Example of reconstruction (dashed line) of an original spectral reflectance function (solid line) obtaining a GFC = 0.9327.

RESULTS

To carry out a detailed study of the goodness of the method of reflectance recovery, we will make this analysis independently for each one of the hue groups in which we have classified our samples.

Red Hue Group

The 1080 sample components of this hue group present, in general, the typical stepped profile that characterizes these types of spectral reflectance curves: low contribution of the short wavelengths, high contribution of the long wavelengths. The results obtained in the recovery of the reflectances in this group are given in Tables I and II.

The first presents the average GFC values reached with the two bases (hue and overall) and different numbers of vectors. Analyzing these values, we find that by using the first 4 vectors of the hue basis, then, on the average, the recoveries have a good level (GFC = 0.9437) and, when we use one more vector, the level rises to very good (GFC = 0.9620); to reach these results with the overall basis, we would need, on the average, 6 (GFC = 0.9553) and 7 vectors (GFC = 0.9615), respectively.

The average information shown in Table I must be complemented in such a way that we can study how the goodness of the recovery is distributed in each and all of the samples constituting the hue group. This is the intent in Table II, which represents the percentage of the recoveries that exceed a series of critical values of GFC using the different bases and different numbers of vectors.

From this table, we can see that, on using the first 5

TABLE I. Averaged GFC values obtained in every hue group with each hue basis (HB) and with the overall basis (OB) for different numbers of vectors.

	Reds		Yellows		Greens		Blues		Purples	
Vct	OB	HB	OB	HB	OB	HB	OB	HB	OB	HB
3	0.8660	0.9191	0.9055	0.9138	0.8228	0.8529	0.8778	0.9008	0.8698	0.9034
4	0.9021	0.9437	0.9318	0.9407	0.8803	0.9155	0.9044	0.9421	0.8920	0.9452
5	0.9360	0.9620	0.9513	0.9635	0.9181	0.9510	0.9387	0.9548	0.9517	0.9536
6	0.9553	0.9785	0.9717	0.9783	0.9446	0.9674	0.9559	0.9678	0.9581	0.9656
7	0.9615	0.9831	0.9732	0.9860	0.9529	0.9745	0.9633	0.9753	0.9653	0.9717
8	0.9695	0.9862	0.9802	0.9888	0.9654	0.9782	0.9691	0.9818	0.9646	0.9810
9	0.9786	0.9892	0.9842	0.9909	0.9730	0.9850	0.9714	0.9865	0.9780	0.9862
10	0.9856	0.9925	0.9885	0.9942	0.9796	0.9893	0.9771	0.9910	0.9840	0.9894

vectors of the hue basis, the immense majority (91.48%) of the recoveries exceed a GFC value of 0.925, and that all the recoveries with the first 7 vectors exceed a GFC of 0.950. If we study the results obtained with the overall basis, we find that by using the first 5 vectors, 71.76% of the recoveries present a GFC that exceeds the value 0.925, and that with 7 vectors, 75.56% of the recoveries exceed a GFC of 0.950.

Yellow Hue Group

The 1062 samples making up this hue group also present the typical stepped profile similar to the red hue group: low contribution of the short wavelengths, high contribution of the medium and long ones. Given the qualitative similarity between these two groups, the results in the recoveries are also similar.

As Table I shows, using the first 5 vectors of the hue basis, we obtain an average GFC of 0.9635; using the first 5 of the overall basis, we obtain an average GFC value of 0.9513.

With respect to percentages, Table II indicates that when the first 5 vectors of the hue basis are used, 93.31% of the recoveries exceed a GFC of 0.925, and with 2 vectors more, all the recoveries surpass this value and practically all (99.62%) have a value greater than 0.950. With regard to the overall basis, with its first 7 vectors, 99.25% of the samples exceed the value 0.925 and 92.94% of these attain a GFC of more than 0.950.

Green Hue Group

The spectral reflectance curves of the 1208 samples comprising this hue group in general present a bell profile typical of these types of functions: low contribution of

TABLE II. Percentage of reconstructed curves that overcome a certain GFC value using different numbers of vectors of the overall (OB) and the hue basis (HB).

			Number of Vectors							
			3	4	5	6	7	8	9	10
GFC > 0,925	Reds	OB	12.87	34.07	71.76	91.96	94.81	98.61	100.00	100.00
		HB	60.00	77.13	91.48	100.00	100.00	100.00	100.00	100.00
	Yellows	OB	43.41	64.03	85.50	97.47	99.25	100.00	100.00	100.00
		HB	54.43	74.11	93.31	99.34	100.00	100.00	100.00	100.00
	Greens	OB	23.12	38.19	54.93	77.30	82.60	93.54	96.85	100.00
		HB	24.11	51.70	84.09	96.69	99.42	99.59	100.00	100.00
	Blues	OB	43.74	53.34	76.39	85.04	89.28	91.53	92.85	95.95
		HB	45.72	70.37	75.07	83.54	85.98	88.52	99.62	100.00
	Purples	OB	30.60	40.10	86.04	91.38	95.78	96.90	98.88	100.00
		HB	50.43	80.78	88.02	94.57	96.55	99.74	99.83	99.91
GFC > 0,950	Reds	OB	0.83	13.43	39.26	66.39	75.56	87.78	98.15	99.91
		HB	23.15	47.59	78.52	96.70	100.00	100.00	100.00	100.00
	Yellows	OB	17.98	39.74	60.08	92.56	92.94	98.17	98.87	99.91
		HB	23.35	48.96	63.28	96.61	99.62	99.81	99.81	100.00
	Greens	OB	12.59	22.20	38.61	58.99	66.45	79.95	92.29	98.84
		HB	9.86	30.49	65.20	88.98	97.10	97.60	99.50	100.00
	Blues	OB	15.80	35.56	62.84	71.87	76.29	82.97	87.80	89.56
		HB	30.10	51.08	62.37	72.44	78.46	84.67	87.30	100.00
	Purples	OB	11.29	30.26	61.72	74.40	85.34	88.62	96.72	99.66
		HB	26.47	50.35	63.45	82.24	90.00	97.76	99.14	99.74

short and long wavelengths, high contribution of the medium.

With the first 5 vectors of the hue basis (Table I), we obtain an average GFC value of 0.9510, though we need the first 7 of the overall basis in order to reach a similar value (0.9529). As we will see below, when we compare these results with their equivalents in the red and yellow hue groups, we conclude that, on the average, we need 1 or 2 vectors more from each linear model in the green hue group in order to match the results obtained with these models in the other two groups.

The study of the percentages in the recoveries (Table II) indicates that, with 5 vectors of these bases, the percentage of recoveries that go above a GFC value of 0.925 is 84.09% and 54.93% for the hue and overall basis, respectively. When we use 7 vectors, the percentages rise to 99.42% and 82.60%, respectively; if we choose to study the relative number of recoveries that exceed a GFC of 0.950, the values fall to 97.10% and 66.45%, respectively. These values are notably lower than those obtained in the two previous hue groups. This reflects the greater difficulty in using a linear model to recover curves with typical bell profiles than to recover profile curves of a stepped type.

Blue Hue Group

The profile of the 1064 spectral reflectance curves that make up this group present, in general, the following features: high contribution of the short wavelengths, relatively important contribution of the medium wavelengths, and null contribution of the long wavelengths.

Tables I and II also show the results obtained in the recovery of reflectances in this group. With the first 5 vectors of the hue basis, we obtain, on the average, a GFC of 0.9548; to obtain similar results, we require one vector more of the overall basis (GFC = 0.9559).

With respect to the percentages in the recoveries, we find the greatest equality of results: with 7 vectors, the percentage of the samples that exceed a GFC of 0.9250 was 85.98% and 89.28%, if we use, respectively, the hue or overall basis; however, if we examine the percentages that rise higher than a GFC of 0.950 with the same number of vectors, we obtain 78.46% and 76.29%, respectively.

Purple Hue Group

This group is composed of 1160 samples, the reflectance curves of which present the classical profile of this type of function: high contribution of short and long wavelengths and low contribution of the medium wavelengths.

Tables I and II show that 5 vectors of these bases (hue and overall) reach average GFC values of 0.9536 and 0.9517, respectively, these values rising to 0.9717 and 0.9653 with the first 7 vectors.

The table of percentages reveals that, from 5 vectors, the two bases obtain similar percentages: with 5 vectors, 88.02% and 86.04% of the recoveries exceed a GFC value of 0.925; with 7 vectors, these percentages rise to 96.55% and 95.78% for the hue and the overall basis, respectively.

DISCUSSION

When preparing a large gamut of samples to generate a linear basis for representing spectral reflectance functions, it is exceptionally advantageous to use acrylic paints because of their versatility for mixing, the temporal stability of their chromatic properties, and the complete equality found between the shape of the spectral reflectance functions of this kind of paints and natural samples.

We obtained the linear basis for every hue group that we clustered for our 5574 samples through principal component analysis. In the same way, we also obtained the overall basis for the complete set. We can state that generating a linear basis for each hue group enables us to improve the results in the recovery by 1 or 2 vectors less than with an overall basis for all the groups. For example, using 5 vectors of each hue basis, the average GFC of the recoveries in each group exceeds 0.950; to obtain the same result in all the groups with the overall basis, we need to use its first 7 vectors. (The overall basis works very well in the yellow hue group, but shows clear limitations in comparison with the green hue group.)

The use of 7 vectors of the overall basis assures that, in the worst of the cases (green hue group), more than 65.5% of the recoveries will go beyond a GFC value of 0.950, exceeding 92% of the recoveries that this value presents in the most favorable of cases (yellow hue group). Therefore, we can affirm that, with 7 vectors of the overall basis, we are assured a more than good recovery of any of our samples.

In this aspect, our results appear to agree qualitatively with those obtained by Parkkinen *et al.*[5] with Munsell samples, who stated that they needed an average of 6–8 vectors of their basis for an adequate recovery of the spectral reflectance curves of the samples studied. Nevertheless, we should take into account that the study of the goodness of their fit focused primarily on the reproduction of the chromaticity coordinates of the original obtained from the recovered reflectance curve. With reference to comparison between functions, they introduced the concept of error bands—the difference between the original curve and the reconstructed for all the wavelengths of the spectrum.

In this sense, our study, with the definition of our goodness-fitting coefficient, is substantially more rigorous, since treating only the differences in absolute values between the original curve and the recovered one may not take into account the real difference between these two curves. Let us imagine two curves corresponding to the spectral reflectance functions of very luminous samples (high spectral-reflectance values), original and recovered, which differ in absolute value by 5 units over the entire spectrum. Now, we take another two curves that also differ in absolute value by 5 units over the entire spec-

trum, but these curves corresponding to samples of low luminance (low spectral-reflectance values). In both cases, the absolute difference between the two pairs of curves is the same, but, in the second case, the real difference between the curves is notably greater.

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