Color-signal filtering in the Fourier-frequency domain

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We have analyzed the Fourier-frequency content of spectral power distributions deriving from three types of illuminants (daylight, incandescent, and fluorescent) and the color signals from both biochrome and nonbiochrome surfaces lit by these illuminants. As far as daylight and the incandescent illuminant are concerned, after filtering the signals through parabolic (low-pass) filters in the Fourier-frequency domain and then reconstructing them, we found that most of the spectral information was contained below 0.016 c/nm. When fluorescent illuminants were involved, we were unable to recover either the original illuminants or color signals to any satisfactory degree. We also used the spectral modulation sensitivity function, which is related to the human visual system's color discrimination thresholds, as a Fourier-frequency filter and obtained consistently less reliable results than with low-pass filtering. We provide comparative results for daylight signals recovered by three different methods. We found reconstructions based on linear models to be the most effective.

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1. INTRODUCTION

A color signal[1] can be defined as any function that represents the spectral power distribution (SPD) of a direct source of light or the product of the spectral reflectance or transmittance of an object and the SPD of the light source that illuminates it. In either case we are dealing with a function that depends upon wavelength. The Fourier transform of this kind of function is another function, the dependent variability of which is known as chromatic frequency.[2] From now on, we will use "F frequency" (frequency in the Fourier domain, as used by Bonnardel and Maloney[3]) to avoid confusion with the electromagnetic frequency. This term is analogous to the terms spatial frequency and temporal frequency, which define the variables obtained by making a Fourier transform of signals that depend upon either space or time, respectively.

The units of F frequency are cycles per nanometer (c/nm), although in earlier studies on this subject, cycles/300 nm[3–8] or cycles/400 nm[9–11] have been used to provide a clearer picture of the significance of the F frequencies. Thus the color signal represented in Fig. 1 corresponds to an SPD with an F frequency of 1 cycle/300 nm (1 c/300 nm). The general mathematical equation for this type of signal is

$$E(\lambda) = E_0[1 + m \sin(2\pi f\lambda + \phi_0)], \qquad (1)$$

where f is the F frequency (expressed in cycles/nanometer), ϕ_0 is the initial phase (expressed in radians), and m is the relative amplitude [with values within the range (0, 1)]. If the function represented in Fig. 1 is infinitely long, its F frequency content corresponds to the frequencies 0 and 1 c/300 nm.

In studies of the F-frequency content of color signals, various authors have shown that the SPDs corresponding either to daylight measurements[3] or, separately, to the spectral reflectances (defined within the visible spectrum) of a wide variety of objects,[3,12,13] can be considered approximately as being F-frequency-limited functions, that is, their F-frequency content drops to a frequency (limiting frequency) at which the contribution of the transform of higher frequencies can be regarded as nil. This, in fact, is never strictly the case, as SPDs, spectral reflectances, and color signals in general can be defined only within ranges constrained by wavelength; nevertheless, the contribution of high frequencies is practically negligible.

Maloney[12] has shown that the spectral reflectances of both natural (Krinov's set) and artificial (Munsell samples) objects can be considered to be F-frequency-limited functions with a limiting frequency of between 0.01 and 0.02 c/nm. Van Hateren[13] analyzed the reflectances of 138 natural objects, both biochrome and nonbiochrome, and found that the limiting F frequency for these reflectance sets was 0.02 c/nm, while Bonnardel and Maloney[3] report a limiting F frequency of 0.0133 c/nm for reflectances of biochrome objects and 0.0033 c/nm for daylight SPDs.

Buchsbaum and Gottschalk[14] have depicted the locus of the color signals represented by Eq. (1) within the CIE 1931 chromaticity diagram for different F frequencies and initial phases. Thus, if the x, y coordinates of any color signal are known, it is possible to find a metamer with a defined F frequency, initial phase and relative amplitude. This implies that any spectral light has a metamer with only low F-frequency content in its Fourier transform. This metamer is an optimum representation of the SPD from a perceptual point of view, but does not necessarily provide information about the limiting F frequency of the SPD itself. To study this limiting F frequency, it is necessary to truncate the SPD in the F-frequency domain and look at the recovered signal after the truncating process.

When we try to evaluate the spectral sensitivity functions of the human visual system within the F-frequency

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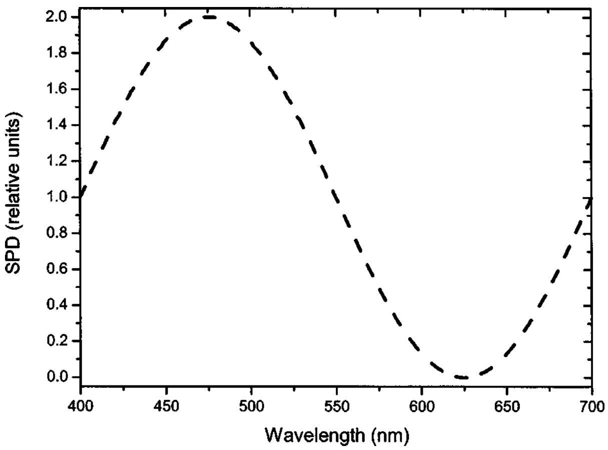

Fig. 1. Example of sinusoidal SPD for $m = 1$, $\phi_0 = 0$, and $f = 1$ c/300 nm (defined between 400 and 700 nm).

domain, we also find that they can be taken to be frequency-limited functions. Thus, in a previous paper,[15] the way in which the color-matching functions of the CIE 1931 standard observer showed limiting frequencies between 0.01 and 0.02 c/nm were described, Barlow[16] had in fact already reported similar results when applying a Fourier transform to cone spectral responses. Those which represent the opponent and nonopponent color-vision mechanisms, obtained in the form of linear combinations of cone responses, must also be F-frequency-limited functions.[3,4,17]

In a manner similar to that in which the contrast sensitivity function in spatial vision is defined to characterize the human visual system as a transmitter of information in spatial frequencies, so in color vision the spectral modulation sensitivity function (SMSF) is defined to characterize the human visual system as a transmitter of information in F frequencies.[5–7,9–11] Analyzing the human visual system in the F-frequency domain is an alternative approach to colorimetry. Sinusoidal SPDs [see Eq. (1)] of a fixed relative amplitude and frequency and varying phase describe an ellipse in the CIE 1931 diagram.[14] It is then possible to obtain chromatic sensitivity thresholds with these kinds of stimulus and analyze them in terms of F frequency and initial phase. This does not imply that the visual system behaves like a filter in the F-frequency domain, as this assumption would imply shift invariance. Shift invariance would mean that the response is insensitive to phase shifts, and this is not the case, since phase changing is equivalent to changing chromaticity along different lines in the CIE 1931 diagram, and it is known that the visual system does not have equal sensitivity to all color variations.

In experiments carried out to determine SMSF[5,6,10,11] with sinusoidal SPD stimuli[6] of different F frequencies and phases or metamers of sinusoidal SPD stimuli,[10] it has been found that the shape of the optimum envelope curve (maximum sensitivity of the different initial phases to each frequency) corresponds to a bandpass filter with a double peak, which is present because of the modulation introduced by each of the two opponent color-vision mechanisms: red–green and yellow–blue. We will show the shape of a typical SMSF later.

Bonnardel and Maloney[3] have demonstrated that there is a certain degree of concordance between the F-frequency limits of color signals obtained with biochrome surfaces under daylight and the limiting F frequency of the SMSF, which they take as 0.016 c/nm.[6] This leads to the belief that for this type of color signal, the visual system processes all or almost all of the useful chromatic information deriving from both the object and the illuminant. Nevertheless, no studies have yet been made into what happens when the object is illuminated by a different kind of light, or what the results of filtering the color signals through other types of filter might be.

An estimation of the F-frequency content of color signals is useful for implementing different methods to recover the signal from few parameters. In previous studies,[12,18] linear methods for spectral and daylight reconstruction have been tested. A complete set of daylight measurements has been successfully reconstructed using principal-component analysis (PCA)[18] to extract a set of orthogonal basis functions from the daylight SPDs. It has been found that a good spectral reconstruction can be obtained with as few as five basis functions. Another method used to achieve a reduction in dimensionality is that of sampling. This method is based on the Shannon–Whitaker theorem, and its performance depends very much on the type of signal reconstructed: Many samples are needed if the signal has narrow peaks (such as a fluorescent illuminant has). Nevertheless, it offers the advantage of not requiring any previous knowledge of a set of the kind of functions that are to be reconstructed. To the best of our knowledge, however, the performance of the linear method for daylight SPDs has not been studied with regard to the reconstruction of signals using sampling and after-filtering in the F-frequency domain.

2. AIMS

The aims of this work were threefold. First, we attempted to analyze in greater depth the supposition that the F-frequency content of color signals can be constrained within certain F-frequency limits. Bonnardel and Maloney's results[3] suggest that the human visual system retains the spectral information for a specific type of signal, but we might ask ourselves what happens when the color signals present information at high chromatic frequencies. This may occur when objects are illuminated by fluorescent sources, the SPDs of which have narrow emission peaks at certain wavelengths within the visible spectrum. The use of parabolic (low-pass) filters allows us to analyze specifically the problem attached to the F-frequency limits of color signals. We have chosen this particular filter function because the Fourier transform of a system with one or more bandpass sensors of different peak wavelengths [such as a color CCD camera or the visual system at the receptoral stage (cone spectral sensitivities)] is roughly a decreasing parabolic function of the F frequency.[3,19] The loss in information that might occur when the high F-frequency content of a color signal is filtered out may or may not lead to filtered signals with significant colorimetric differences from the original signal. When the differences in color are insignificant, we can claim to have frequency-limited functions that satisfactorily represent the original color signals.

Our second aim was to study the effects of making the analysis described above using a real filter such as the SMSF.[10] Such a study, which to the best of our knowledge has never been made before, would give us information concerning the representation of color signals by the human visual system. In this case we used a bandpass SMSF filter that not only filters out the high chromatic frequencies but also some of the lower ones. The shape of the SMSF indicates that the filtering at low frequencies is fairly smooth without resulting in the total elimination of the information at these frequencies. Van Hateren[13] explained the contribution of this kind of filtering to the phenomenon of color constancy. We shall look at his hypothesis in Section 4.

Our third aim was to compare the filtered signals with ones reconstructed by two mathematical methods. The first of these mathematical methods is a linear model consisting of a PCA, the basis of the eigenvectors being obtained from our own measurements.[18] The second involves obtaining an expression for daylight by application of the Shannon–Whitaker theorem, accepting the constraint of a limiting F frequency.[20] This method is similar to the one described by Stiles *et al.*[21] for the spectral reflectance of objects. We set out our results in Subsection 4.C.

3. METHODS

A. Illuminants

We used both natural (daylight) and artificial illuminants. For the daylight illuminants we took a set of 40 measurements made by members of our laboratory[18] at different times of day, some sunny, some cloudy, and some under mixed conditions.

The artificial illuminants used were the incandescent illuminant A and the fluorescent illuminants F2, F7, and F11, the latter being chosen on the basis of recommendations supplied by the CIE.[22] We also added a commercial fluorescent light source the SPD of which we confirmed to be similar to those normally used for interior lighting. The SPDs of the artificial illuminants used, together with some of the daylight measurements, are shown in Fig. 2. All the color signals analyzed were sampled between 400 and 700 nm at intervals of 5 nm.

B. Objects

We studied the 170 spectral reflectances from Vrhel and co-workers' database of natural and artificial objects[23] and found considerable diversity among these groups of data with regard to the kinds of spectral reflectance involved, either in the shape of a step or one or two wide peaks. In this way they covered a wide range of hues: blues, greens, yellows, oranges, reds and purples, both natural and artificial. We divided the objects into two subgroups corresponding to biochrome (96) and nonbiochrome (74) surfaces in view of the possibility that mixing objects the color of which is generated by different physical processes might lead to average results representing no particular category. Added to this, statistically significant differences were found in a within-subject analysis of the results given by both subgroups.

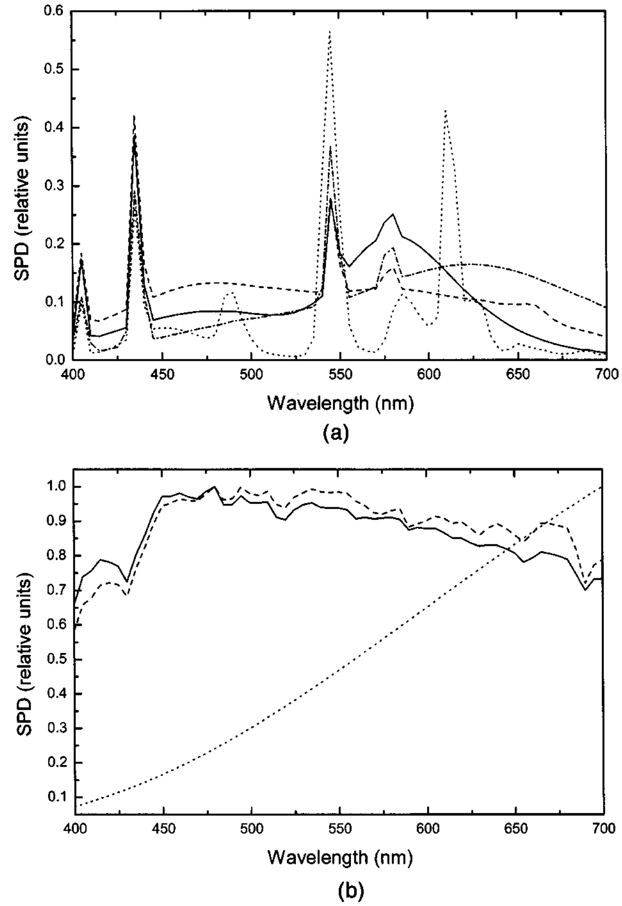

Fig. 2. (a) SPDs of the fluorescent illuminants used: solid curve, F2; dashed curve, F7; dotted curve, F11; dotted-dashed curve, commercial fluorescent. (b) Two examples of daylight SPD and CIE illuminant A: solid curve, daylight 1; dashed curve, daylight 2; dotted curve, A.

C. Procedure

As standard procedure, we took a color signal, either the SPD of an illuminant or the product of the SPD of the illuminant and the spectral reflectance of the object, and, after applying a Hanning (raised cosine) window to reduce spectral leakage due to the finite and discrete nature of the SPDs, we calculated its Fourier transform. We then applied a filter to this transform, either parabolic with different cutoff frequencies, or the SMSF.[10] Subsequently, we made an inverse Fourier transform of the filtered signal in the F-frequency domain; after discounting the Hanning window, we obtained the filtered signal in the spectral domain.

To compare the filtered signal with the original one, we made a double evaluation, both spectral and colorimetric, in accordance with the recommendations of Imai *et al.*,[24] who indicate that there is no single parameter which permits us to evaluate the validity of a spectral reconstruction. Thus our evaluation contained a colorimetric index (CIELAB color difference) and a spectral one [the proportion of the spectral energy (PSE)[12] below an F-frequency limit]. With the former parameter we could establish the acceptability or otherwise of the reconstruction as far as visual perception is concerned, i.e., from the point of view of the observer who has to judge the reconstruction. With the PSE index, we took into account only the physical aspects of the reconstruction, i.e., the differences in value between the original and filtered signals for each wavelength. This index therefore indicates the mathematical quality of the reconstruction from a spectral viewpoint. Neither of the indices on its own is able to contribute enough information to establish the closeness of the filtered signals to the originals, but each does address one of the basic aspects of the problem, the spectral or the colorimetric, so we consider their combined use to be adequate.

We also used both the spectral metric (PSE) and the perceptual metric (CIELAB color difference) to compare daylight SPDs reconstructed by linear methods (PCA), sampling before and after parabolic filtering of different cutoff frequencies. The PCA method[25] finds the best fitting set of basis functions corresponding to a "zero-centered" PCA (according to Brill's terminology[26]) of a set of 2600 daylight SPDs.[18]

As we mentioned in Subsection 3.B, daylight has a limiting F frequency, which leads us to believe that it might also admit of mathematical representations based on the Shannon–Whitaker theorem:

$$E(\lambda) = \sum_{n=0}^{N} E\left(\frac{n+m}{2f_l}\right) \mathrm{sinc}\left[2f_l\left(\lambda - \frac{n+m}{2f_l}\right)\right], \quad (2)$$

where f_l is the limiting F frequency and $[m, m+N]$ is the positive integers's interval corresponding to the limits of the visible spectrum. Thus if we are considering a limiting F frequency of 0.01 c/nm, we would take signal values at intervals of 50 nm, which, within the spectral range in question, would mean $N = 6$ and $m = 4$ in Eq. (2), a total of seven samples of $E(\lambda)$ for the SPD.

4. RESULTS

A. Parabolic Filtering

In Tables 1–6 we set out the results for the PSE and CIELAB color differences obtained on comparing the original signals with those filtered through low-pass parabolic filters with various cutoff frequencies chosen in the following way: Frequencies 0.013 and 0.02 c/nm derive from a previous study in which, according to various different chromatic discrimination criteria, we found them to be the limiting frequencies for the human visual system[9]; an F frequency of 0.016 c/nm is that indicated by Bonnardel and Maloney as the limit for the product of color signals from objects in daylight,[3] which coincides with that measured by Bonnardel *et al.*[6] for the SMSF;

Table 1. PSE below a Frequency Limit for Color Signals Corresponding to SPDs of the Different Illuminants[a]

Frequency (c/nm)	Illuminant					
	Daylight	Illuminant A	Commercial Fluorescent	F2	F7	F11
0.013	0.9957 [0.9952, 0.9963]	0.9924	0.8697	0.8704	0.8729	0.4382
0.016	0.9966 [0.9961, 0.9970]	0.9964	0.8719	0.8748	0.8740	0.4962
0.02	0.9973 [0.9969, 0.9981]	0.9996	0.8726	0.8751	0.8744	0.5666
0.04	0.9981 [0.9976, 0.9989]	0.9999	0.9175	0.9081	0.9052	0.7817

[a] Values for daylight are mean values of 40 daylight SPDs; tenth and ninetieth percentiles are shown in brackets.

Table 2. CIELAB Color-Difference ($\Delta E_{\mathrm{CIELAB}}$) Values on Comparing Original and Filtered Color Signals by Use of Parabolic Filters with Different Cutoff Frequencies[a]

Frequency (c/nm)	Illuminant					
	Daylight	Illuminant A	Commercial Fluorescent	F2	F7	F11
0.013	3.23 [2.14, 5.11]	2.79	14.56	21.92	19.59	60.56
0.016	2.96 [2.12, 4.21]	1.27	14.36	21.70	19.47	49.54
0.02	2.37 [1.34, 3.39]	0.68	14.40	21.27	19.23	44.05
0.04	0.52 [0.37, 0.92]	0.15	12.22	19.32	16.78	29.17

[a] Values for daylight are mean values of 40 daylight SPDs; tenth and ninetieth percentiles are shown in brackets.

Table 3. Mean Values of PSE below a Frequency Limit for 96 Biochrome Surfaces with Three Illuminants[a]

Frequency (c/nm)	Illuminant		
	Daylight	Illuminant A	Commercial Fluorescent
0.013	0.9845 [0.9615, 0.9944]	0.9654 [0.8998, 0.9889]	0.9176 [0.8332, 0.9619]
0.016	0.9915 [0.9753, 0.9981]	0.9879 [0.9490, 0.9995]	0.9217 [0.8507, 0.9609]
0.02	0.9928 [0.9805, 0.9982]	0.9906 [0.9624, 0.9999]	0.9256 [0.8569, 0.9634]
0.04	0.9946 [0.9897, 0.9984]	0.9968 [0.9846, 0.9999]	0.9545 [0.9095, 0.9801]

[a] The tenth and ninetieth percentiles are shown in brackets.

Table 4. Mean Values of the PSE below a Frequency Limit for 74 Nonbiochrome Surfaces with Three Illuminants[a]

Frequency (c/nm)	Illuminant		
	Daylight	Illuminant A	Commercial Fluorescent
0.013	0.9891 [0.9761, 0.9968]	0.9871 [0.9690, 0.9919]	0.8899 [0.7784, 0.9682]
0.016	0.9926 [0.9832, 0.9976]	0.9934 [0.9775, 0.9993]	0.8910 [0.7795, 0.9711]
0.02	0.9945 [0.9874, 0.9980]	0.9972 [0.9907, 0.9999]	0.8954 [0.7898, 0.9725]
0.04	0.9963 [0.9927, 0.9985]	0.9994 [0.9885, 0.9999]	0.9343 [0.8661, 0.9626]

[a] The tenth and ninetieth percentiles are shown in brackets.

Table 5. Mean CIELAB Color-Difference ($\Delta E_{\mathrm{CIELAB}}$) Values on Comparing Original and Filtered Color Signals by Use of Parabolic Filters with Different Cutoff Frequencies for 96 Biochrome Surfaces with Three Illuminants[a]

Frequency (c/nm)	Illuminant		
	Daylight	Illuminant A	Commercial Fluorescent
0.013	2.44 [1.08, 3.97]	3.00 [1.63, 4.31]	7.83 [1.76, 17.63]
0.016	1.39 [0.49, 2.69]	2.39 [1.29, 3.40]	7.72 [1.33, 17.42]
0.02	1.00 [0.26, 2.02]	2.21 [1.25, 3.04]	7.45 [0.97, 19.10]
0.04	0.42 [0.05, 1.04]	0.67 [0.38, 0.94]	6.21 [0.57, 14.62]

[a] The tenth and ninetieth percentiles are shown in brackets.

and last, with the use of 0.04 c/nm, our intention was to widen the range of F frequencies studied.

To evaluate our results, we had to establish criteria for the minimum value of PSE and the maximum for the CIELAB color difference. Only in this way could we establish the spectral and/or colorimetric validity of the reconstructions obtained after filtering the signals. As far as PSE is concerned, in accordance with Bonnardel and Maloney,[3] we have taken the reconstruction to be acceptable if this parameter is equal to or more than 0.9900, which is equivalent to accepting a maximum loss of 1% in the energy of the filtered signal. As for color difference, we have followed the opinions of Vrhel *et al.*[23] and Finlayson,[27] who agree that differences of less than 3 CIELAB units between the reconstructed signal and the original are acceptable, although this limit might be considered somewhat generous given that 0.5–1.0 CIELAB units have been used for measurement standards.[28]

The PSE and $\Delta E_{\mathrm{CIELAB}}$ values for color signals deriving from the SPD of the illuminants are set out in Tables 1 and 2. In the case of daylight, the values in the tables are means of the 40 SPDs analyzed. With regard to our criteria of acceptability, it can be seen that for daylight we get filtered signals with a limiting F frequency of 0.016 c/nm or above, which we can accept as satisfactory reconstructions of the original signals. Even for 0.013 c/nm, we can accept the validity of the spectral reconstruction, but perhaps not the colorimetric one. Bonnardel and Maloney[3] suggest that the acceptable limiting F frequency for daylight is 0.0033 c/nm with reference to the spectral criterion alone. They make no kind of colorimetric evaluation of this type of signal, although they do for spectral reflectances of objects. We studied a parabolic filter with this cutoff F frequency, and found that, in this case, the average PSE value and the CIELAB color difference were 0.9908 and 5.41, respectively. Our results concerning spectral reconstructions confirm those of Bonnardel and Maloney, but the reconstruction from the colorimetric point of view is not good enough with filters with this cutoff F frequency.

In Fig. 3 we show an example of a daylight SPD and our reconstruction of it with different limiting frequencies. The signal filtered at 0.016 c/nm retains the general shape of the SPD, although it is smoothed somewhat by the loss of the absorption bands. The same result applies to the SPD filtered at 0.013 c/nm. As might be expected, with frequencies of 0.02 (not shown in the figure) and 0.04 c/nm, we obtained excellent results for both the PSE and color-difference values.

With illuminant A we obtained results similar to those obtained with daylight (Tables 1 and 2). This was to be expected given the continuous, smooth slope of its SPD. Just the opposite was true, however, with the fluorescent illuminants. We obtained no satisfactory reconstructions, either spectral or colorimetric, for any of the signals studied, even at the higher frequencies. What is more, not even in the reconstruction at the limiting frequency of 0.1 c/nm, which is the maximum frequency obtained in the Fourier transform, did we achieve a perfect result. Neither the data provided by the CIE concerning fluorescent illuminants nor the spectral resolution of most spectroradiometers allow us to work with narrower intervals, so we can conclude only that the F-frequency limits for color signals from fluorescent illuminants are higher than 0.1 c/nm, an F frequency far higher than that detected by the human visual system. Thus, we might affirm that in color vision, we lose a lot of the spectral information con-

Table 6. Mean CIELAB Color-Difference ($\Delta E_{\mathrm{CIELAB}}$) Values on Comparing Original and Filtered Color Signals by Use of Parabolic Filters with Different Cutoff Frequencies for 74 Nonbiochrome Surfaces with Three Illuminants[a]

Frequency (c/nm)	Illuminant		
	Daylight	Illuminant A	Commercial Fluorescent
0.013	3.27 [1.17, 6.15]	3.09 [1.16, 5.23]	7.92 [3.00, 18.09]
0.016	1.80 [0.50, 3.89]	2.41 [1.01, 3.86]	7.83 [2.41, 17.84]
0.02	1.28 [0.24, 2.62]	2.22 [1.04, 3.48]	7.63 [2.27, 18.78]
0.04	0.48 [0.05, 1.22]	0.67 [0.32, 0.98]	6.27 [0.60, 16.10]

[a] The tenth and ninetieth percentiles are shown in brackets.

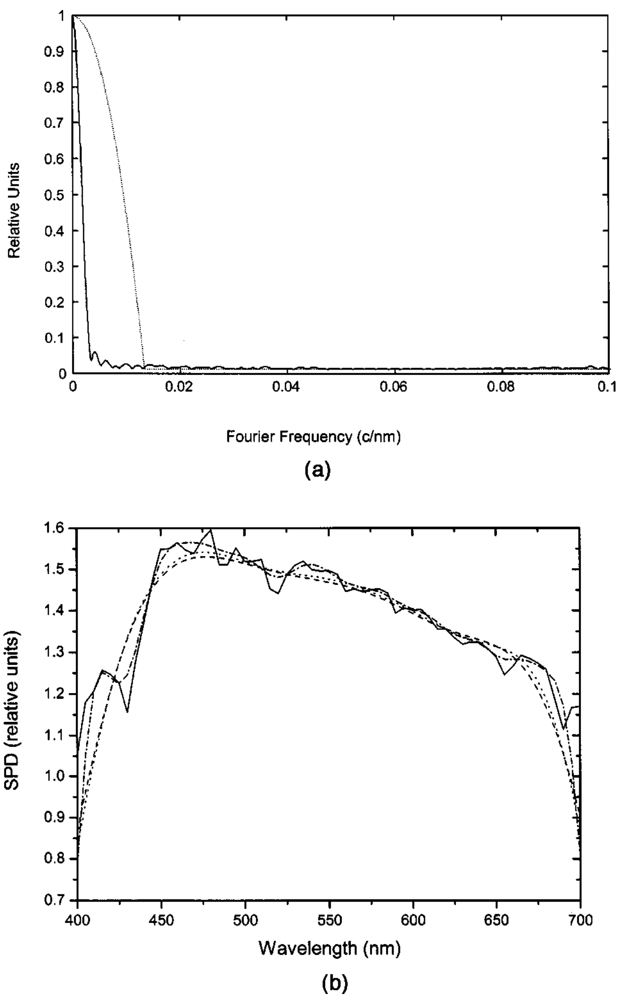

Fig. 3. (a) FFT of daylight SPD and parabolic filter of 0.02 c/nm: solid curve, SPD; dotted curve, parabolic filter. (b) Daylight SPD and three filtered signals obtained with a parabolic filter with three different cutoff frequencies: solid curve, original; dotted-dashed curve, $f = 0.04$ c/nm; dotted curve, $f = 0.016$ c/nm; dashed curve, $f = 0.013$ c/nm.

tained in this kind of signal. In Fig. 4 we show an example of the SPD of a fluorescent illuminant filtered at various cutoff frequencies. As can be seen, the filtered signal has lost practically all the information belonging to the typical emission peaks of signals from fluorescent sources.

An analysis of the color signals is set out in Tables 3–6 for the two subgroups of biochrome and nonbiochrome surfaces described in Subsection 3.B and three different illuminants (daylight, illuminant A and commercial fluorescent). We have found that, in general, biochrome surfaces gave better results for CIELAB color differences and slightly worse results for PSE. For daylight and biochrome surfaces, the PSE for a limiting frequency of 0.016 c/nm is under but very close to 0.99, and colorimetrically the recovered signals are under 3 CIELAB units of color difference from the original signals. For daylight and nonbiochrome surfaces, all the relevant spectral and colorimetric information is contained in the frequencies below 0.016 c/nm. Our overall results for color signals are similar to those obtained by Bonnardel and Maloney,[3] who indicate that the limiting F frequency for color signals resulting from the product of the spectral reflectance of an object and the SPD of the daylight illuminating it is around 0.016 c/nm, which, since this coincides with the limiting frequency of the SMSF,[6] leads us to the conclusion that the human visual system does retain the spectral information of this kind of signal, and thus captures the chromatic information from our natural environment quite efficiently. To arrive at this conclusion, Bonnardel and Maloney[3] took into account only the value of the limiting frequency of the SMSF, not its shape, which we will comment on at greater length in Subsection 4.B.

For illuminant A, our results for color signals are slightly worse concerning the spectral metric, so the cutoff F frequency would be 0.02 c/nm for biochrome surfaces and 0.016 c/nm for nonbiochrome surfaces. Colorimetrically, the cutoff F frequency is 0.016 c/nm for both subgroups. It is also quite feasible that these results might be extended to other illuminants with smoothly sloped SPDs, such as Planckian SPDs (illuminant A is of this type).

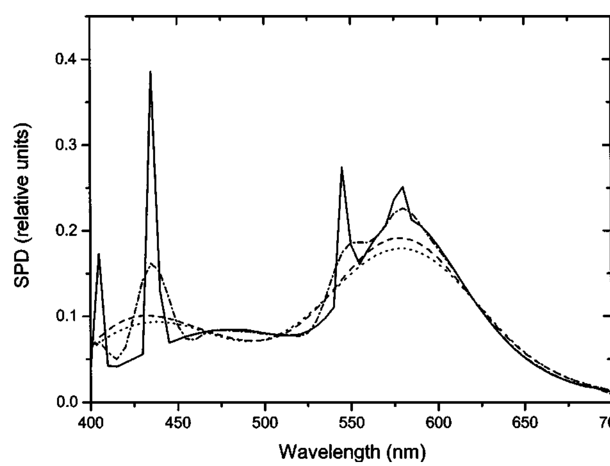

Fig. 4. SPD of the illuminant F2 and three filtered signals obtained with a parabolic filter with three different cutoff frequencies: solid curve, original; dotted-dashed curve, $f = 0.04$ c/nm; dashed curve, $f = 0.016$ c/nm; dotted curve, $f = 0.013$ c/nm.

As we have seen with daylight, small "kinks" in the SPD of the illuminant disappear in the filtered signal. Nevertheless, the sharp emission peaks registered with the fluorescent illuminants do not permit any filtered signal, either spectral or colorimetric, to turn out satisfactorily. For this kind of signal, the loss of information is considerable, whether the filters be low pass, as studied in this subsection, or bandpass, as we shall see in the next subsection. The results for both subgroups of our collection of color signals and the commercial fluorescent illuminant (Tables 5 and 6) turn out to be quite poor, though slightly better for biochrome surfaces than for nonbiochrome. The other fluorescent illuminants gave very similar results. On comparing the results for the color signals from our collection of surfaces and a fluorescent illuminant (Tables 3–6) with the results for the SPD of the fluorescent illuminant (Tables 1 and 2), it can be deduced that the reconstruction is more satisfactory for color signals than for the SPD of the fluorescent illuminant, both spectrally and colorimetrically. This does not happen with the other two illuminants used (daylight and illuminant A), which contain fewer high F frequencies than the fluorescent illuminants. This discrepancy in behavior can be put down to the color signal's (object × illuminant) containing a higher amount of low F frequencies than does the SPD of the illuminant. The convolution theorem[29] states that if a signal is the product of two functions, then its limiting frequency in the F domain must be the sum of the limiting frequency of both functions; but as we have seen with the fluorescent illuminants, we are not able to estimate the limiting frequency because it is higher than the Nyquist frequency corresponding to the sampling of the color signals (0.1 c/nm). Although the limiting frequency of the color signal is higher than that of the SPD of the fluorescent illuminant, the relative content in low F frequencies is higher in the color signal than in the SPD because the reflectance of the object adds content in the low F-frequency range. When we apply a parabolic filter, we are selecting the low F content of the signal; thus the color signal is more completely recovered after filtering than the SPD of the fluorescent illuminant. This is not the case with the other two illuminants (daylight and illuminant A) because they contain fewer high F frequencies and we are able to determine the limiting frequency of the color signals, so the results are what might be expected in terms of the predictions based on the convolution theorem. In other words, the distribution of F frequencies in the color signal is different from that in the SPD of the fluorescent illuminant alone, and this leads to a better recovered signal after low-pass filtering.

B. Spectral Modulation Sensitivity Function Filtering

The signals filtered by the SMSF cannot become incoming signals for the visual system, and this reduces the significance of the calculation of chromaticity coordinates for these filtered signals. It would be like applying a double filtering process with both filters originating in the visual system and one (the SMSF) including the other (photoreceptor's spectral sensitivities). Thus we shall discuss the results for SMSF-filtered signals in terms of PSE and not CIELAB color differences.

When the color signals studied were filtered through the SMSF measured in our laboratory,[10] the resulting filtered signals for the PSE (Table 7) were in no way acceptable from a spectral point of view. We should bear in mind that this filter, owing to its bandpass shape with a double peak, affects not only the high chromatic frequencies but also the medium and low ones. We have also used the SMSF curve obtained by Bonnardel *et al.*[6] as a filter in the F-frequency domain. With Bonnardel and coworkers' SMSF, the results were very similar to those shown in Table 7, the only notable difference being that for the set of 40 daylight SPDs, the mean PSE of the reconstructed signals is 0.9901; that is, more than 99% of the spectral energy has been recovered. Neither the reconstructed signals for the SPDs of the other illuminants used nor those of both sets of color signals (corresponding to biochrome and nonbiochrome surfaces) managed to recover 99% of their energy, though in general the spectral energy recovered was higher than with our SMSF filter (except for the fluorescent F11 and the commercial fluorescent color signals for both sets of reflectances). We attribute these differences mainly to the fact that the decrease in sensitivity toward the cutoff frequency of our SMSF is sharper than that of Bonnardel *et al.*[6]

Some examples of signals filtered through the SMSF are given in Figs. 5(a)–5(c). These signals represent information filtered through a function that takes into account some of the characteristics of the human visual system when it processes color information, though this does

Table 7. PSE on Comparing Original and Filtered Color Signals by Use of SMSF Filter[a]

Signal	PSE
Illuminant	
Daylight	0.9864 [0.9849, 0.9896]
Illuminant A	0.9735
Com. Fluor.	0.8650
F2	0.8719
F7	0.8701
F11	0.5137
Biochrome surface	
Daylight	0.9735 [0.9555, 0.9836]
Illuminant A	0.9328 [0.8668, 0.9731]
Com. Fluor.	0.9409 [0.9120, 0.9641]
Nonbiochrome Surface	
Daylight	0.9791 [0.9658, 0.9885]
Illuminant A	0.9568 [0.9282, 0.9740]
Com. Fluor.	0.8942 [0.7587, 0.9641]

[a] Color signals correspond to SPDs of the illuminants and color signals from 96 biochrome and 74 nonbiochrome surfaces with three different illuminants. Values for daylight are mean values of 40 daylight SPDs. The tenth and ninetieth percentiles are shown in brackets. Com. Fluor. stands for commercial fluorescent.

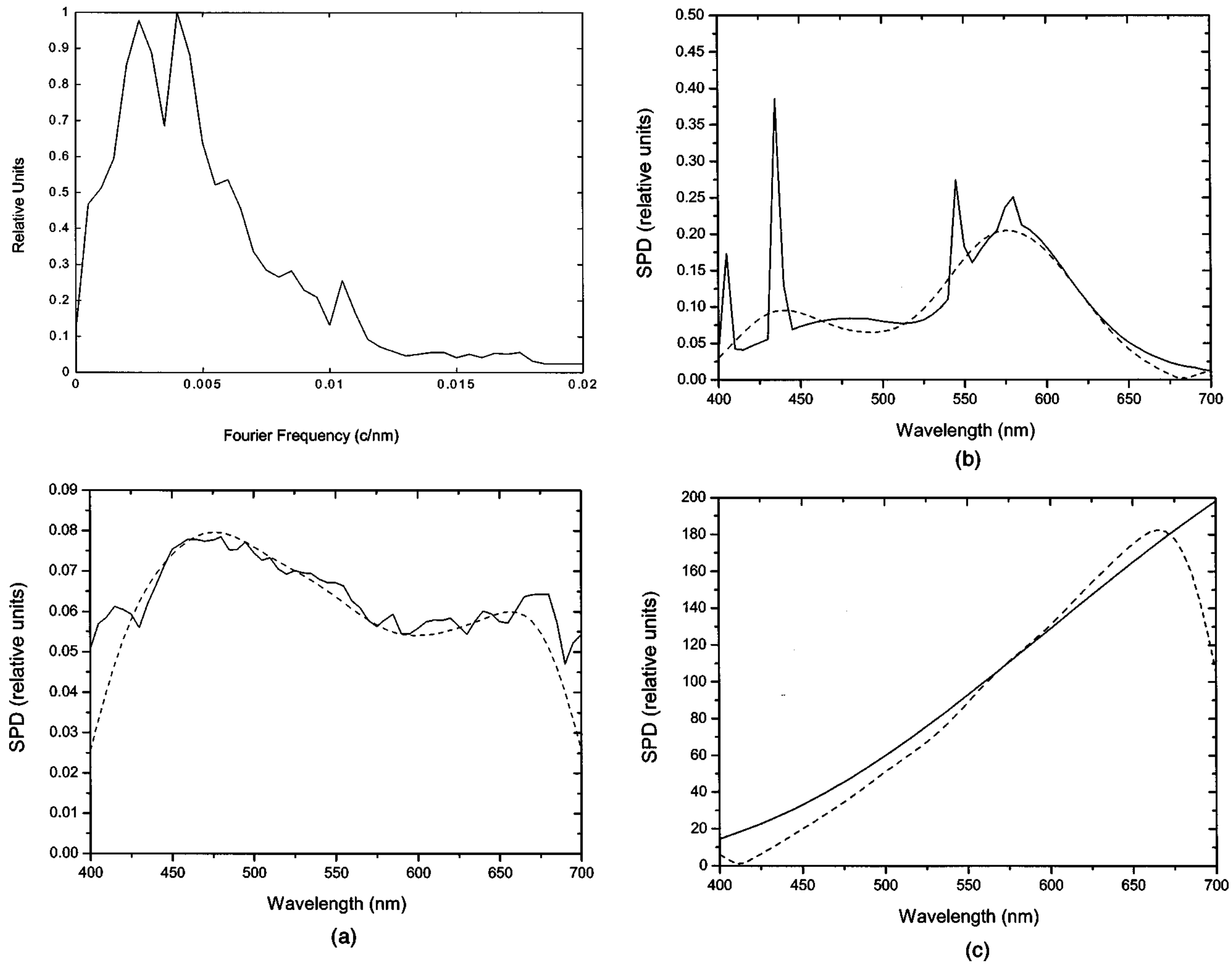

Fig. 5. Original (solid curve) and filtered (dashed curve) signals obtained with the SMSF filter: (a) daylight; upper panel, normalized SMSF filter (see Ref. 10); (b) F2 fluorescent; (c) illuminant A.

not imply that the human visual system acts as a filter in the F-frequency domain, because it is not invariant to phase shifts. The filtered signals are quite smooth, the high chromatic frequencies having been eliminated. The filtering process with the SMSF results in the loss of more than 1% of the energy generated by daylight and even higher losses when the objects are illuminated by conventional artificial light sources.

The filtering of low frequencies requires some additional comments. Van Hatteren[13] demonstrated the contribution to color constancy of the human visual system's filtering of low F frequencies. He assumed that the different information provided by the illuminants is contained within the low frequencies, thus filtering these frequencies from the color signal will help to divorce the perception of the object from whatever illuminant might be used (color constancy). We have tested this hypothesis by making a direct comparison between the filtered color signals of a single object viewed under different illuminants (Fig. 6). Considerable spectral differences can be observed between filtered signals. From this figure we can deduce that a filtering process in the F-frequency domain is not enough to remove the illuminant information from color signals.

C. Daylight Representation

The mathematical representation of daylight is of considerable importance in the analysis of color images.[30,31] Since the work of Judd *et al.*,[32] this representation has often been made on the basis of a linear model containing very few parameters. In a previous study,[18] we described how, with three to five orthogonal vectors from a linear base, it was possible to represent a wide set of daylight SPDs quite satisfactorily. Although our results turned out to be very acceptable, it is still worthwhile asking whether other representations might not be possible on the basis of daylight as an F-frequency-limited function with a determined limiting frequency.

To study the validity of this representation of daylight we compared the three methods of representation we have investigated. From one of our original sets of measurements, we made a mathematical reconstruction by using a linear model containing very few vectors (two and three) with Eq. (2) and different limiting frequencies, i.e., at different sampling intervals of the signal, and by applying parabolic filters with different cutoff frequencies. The graphs thus obtained are shown in Fig. 7. To make a numerical comparison we used the PSE and the CIELAB color difference.

We obtained the best representations with the linear model [Fig. 7(a)]. Only two (PSE = 0.9992, $\Delta E_{\mathrm{CIELAB}}$ = 1.43) or three (PSE = 0.9999, $\Delta E_{\mathrm{CIELAB}}$ = 0.17) previously deduced, linear-based vectors[18] were enough to obtain more satisfactory results than with the other representations. Nevertheless, it should be pointed out that it is also possible to arrive at valid daylight representations with only a few parameters by using the Shannon–Whitaker theorem. In Fig. 7(b) we show the reconstructions obtained by using Eq. (2) and taking limiting frequencies of 0.005, 0.01 and 0.02 c/nm, which is the same as taking 4, 7, and 13 terms, respectively, or equally spaced samples between 400 and 700 nm in the SPD. Although the results for four samples were satisfactory as far as the spectral reconstruction was concerned (PSE = 0.9967), they were not so with regard to the colorimetric reconstruction ($\Delta E_{\mathrm{CIELAB}}$ = 5.90). Nevertheless, for seven samples (PSE = 0.9968 and $\Delta E_{\mathrm{CIELAB}}$ = 2.15), we obtained a good representation of daylight according to both criteria.

The values for the indices used for the filtered signals in Fig. 7(c) are similar to those in Fig. 7(b), as might be expected. Thus the signal filtered at 0.02 c/nm shows values (PSE = 0.9987 and $\Delta E_{\mathrm{CIELAB}}$ = 1.32) close to those obtained by means of Eq. (2) with 13 samples (PSE = 0.9984 and $\Delta E_{\mathrm{CIELAB}}$ = 0.90). We also obtained results with seven samples, (which is the same as assuming a limiting frequency of 0.01 c/nm) similar to those with the signal filtered at 0.013 c/nm (PSE = 0.9973 and $\Delta E_{\mathrm{CIELAB}}$ = 2.92) and at 0.016 c/nm (PSE = 0.9984 and $\Delta E_{\mathrm{CIELAB}}$ = 1.99). Only small increases in the color-difference values are found for the filtered signals.

Our results give us analytical equations for daylight as the sum of a few terms of well-known functions (sinc). To this end, we have to know the value for daylight in a specific but not very wide number of wavelengths. This requires nothing more than the measurement of the SPDs of daylight at certain equally spaced wavelengths (seven samples, for example) in the visible spectrum to obtain a satisfactory spectral and colorimetric representation.

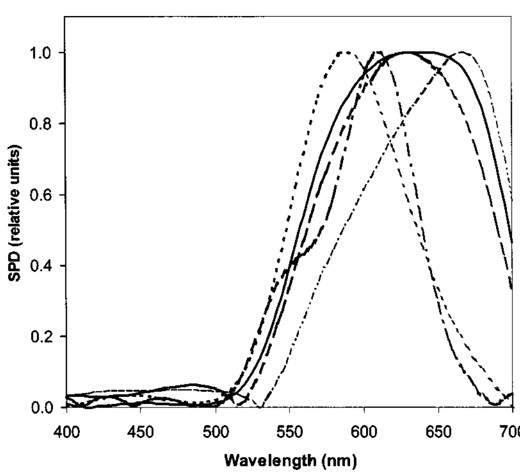

Fig. 6. Normalized SMSF-filtered signals for object v142 (see Ref. 23) and five different illuminants: solid curve, daylight; long dashed, commercial fluorescent; short dashed, F1; heavy dotted-dashed, F10; light dotted-dashed, A.

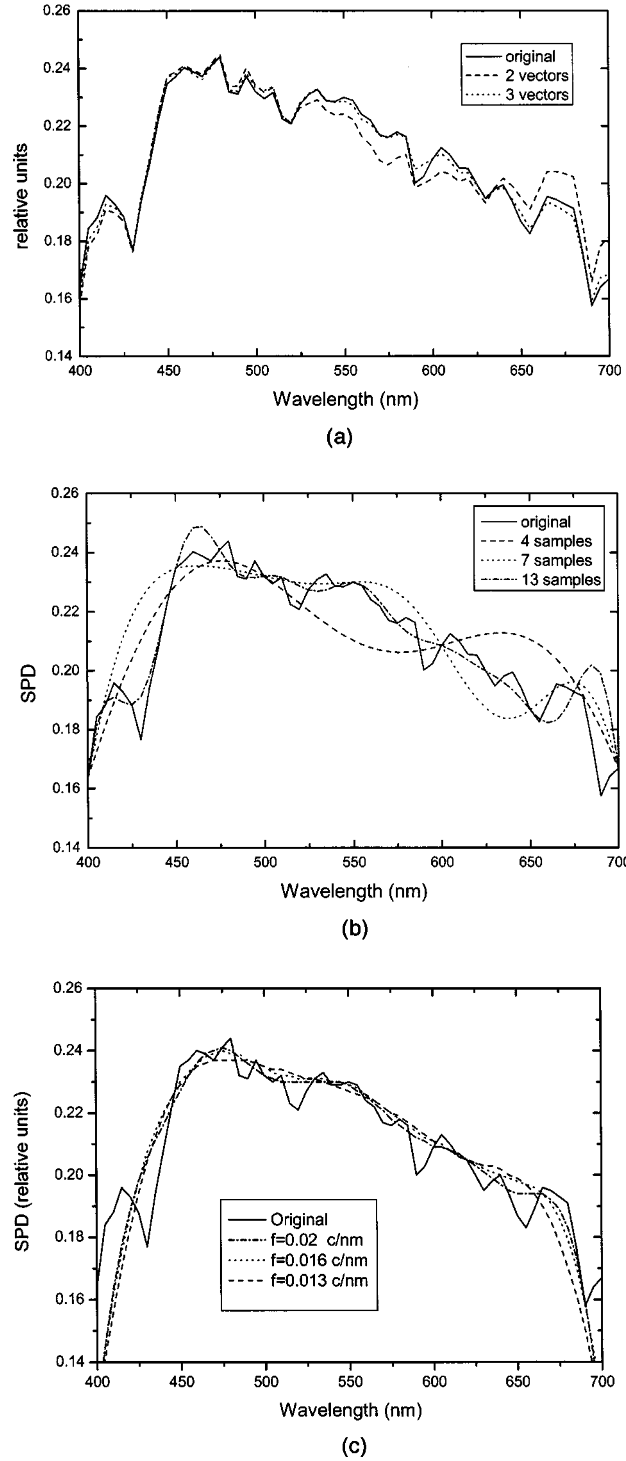

Fig. 7. Original and reconstructed daylight signals: (a) linear model of varying number of vectors, (b) Eq. (2) with different sampling intervals, (c) parabolic filter of three different cutoff frequencies.

5. CONCLUSIONS

An analysis of the color signals in the F-frequency domain provides information for a new approach to the processing of color signals, as the systems for capturing and reproducing color images can be looked upon as transmitters of information within F frequencies. Within this domain, we have analyzed color signals deriving from both biochrome and nonbiochrome surfaces illuminated by daylight, incandescent and fluorescent illuminants. After passing the signals through parabolic (low-pass) filters with various cutoff frequencies, we evaluated them both from a spectral and colorimetric point of view and came to the conclusion that, as far as daylight and the incandescent illuminant are concerned, most of the relevant information in these signals is contained within frequencies below 0.016 c/nm. When, on the other hand, fluorescent illuminants are involved, a considerable amount of information is contained within higher frequencies and, with the parabolic filters of different cutoff frequencies used, we were unable to obtain signals comparable to the originals.

When we studied the filtering process by the SMSF, our results indicated that in the signals, there was a clear loss in the absorption bands of the atmospheric components of daylight. As might be expected, this bandpass filtering smoothed out the slopes of the color signals. Apart from this, the differences between types of illuminant are not reflected in the content of the lower frequencies.

Our results support those previously published by Bonnardel and Maloney[3] in which they propose a concordance between the limiting F frequency of color signals illuminated by daylight and that of the human visual system. Our results also extend these findings to include the incandescent light source used in our experiments.

We also compared three ways of reconstructing daylight: by filtering in the F-frequency domain; by applying Eq. (2) as provided by the Shannon-Whitaker theorem; and by a linear model containing some few vectors which we had obtained in a previous study.[18] This last method provided us with the best results because it required only two or three eigenvectors from the base to achieve a satisfactory reconstruction of daylight, both in spectral and colorimetric terms. Nevertheless, if the value of the SPD is known for some few equidistant wavelengths, it is possible with Eq. (2) to arrive at an analytical representation of daylight in the form of the sum of a few terms of well-known functions. This finding may be of great interest in constructing algorithms for analyzing and synthesizing color images in artificial color vision.

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